Wisconsin Teacher of Mathematics
Fall 2017

The Wisconsin Teacher of Mathematics, the official journal of the Wisconsin Mathematics Council, is published twice a year. Annual WMC membership includes a 1-year subscription to this journal and to the Wisconsin Mathematics Council Newsletter. The Wisconsin Teacher of Mathematics is a forum for the exchange of ideas. Opinions expressed in this journal are those of the authors and may not necessarily reflect those of the Council or editorial staff.

The Wisconsin Teacher of Mathematics welcomes submissions for the Fall 2017 issue. We encourage articles from a broad range of topics related to the teaching and learning of mathematics. In particular, we seek submissions that:

• Present engaging tasks that can be implemented in the Pk–12 classroom.
• Connect mathematics education research and theory to classroom practice.
• Showcase innovative uses of technology in the classroom.
• Focus on work with preservice teachers in the field.
• Discuss current issues or trends in mathematics education.

Other submissions not focusing on these strands are also welcome. If you have questions or wish to submit an article for review, please email Josh Hertel (jhertel@uwlax.edu) or visit the WMC website for more information (http://www.wismath.org/Write-for-our-Journal). The submission deadline for the Fall 2017 issue is August 31, 2017.

Manuscript Submission Guidelines

• Manuscripts may be submitted at any time.
• All manuscripts are subject to a review process.
• Include the author’s name, address, telephone, email, work affiliation and position.
• Manuscripts should be double-spaced and submitted in .doc or .docx format.
• Embed all figures and photos.
• Send an electronic copy of the manuscript to jhertel@uwlax.edu.

The National Council of Teachers of Mathematics selected the Wisconsin Teacher of Mathematics to receive the 2013 Outstanding Publication Award. This prestigious award is given annually to recognize the outstanding work of state and local affiliates in producing excellent journals. Judging is based on content, accessibility, and relevance. The WMC editors were recognized at the 2014 NCTM annual meeting.
# Table of Contents

Volume 69 | Number 1 | 2017 Special Issue

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Editorial</td>
<td>1</td>
</tr>
<tr>
<td>President's Message: The Gift of Giving in Mathematics Education</td>
<td>2</td>
</tr>
<tr>
<td>Sister Mary Petronia Van Straten: A Short Biography</td>
<td>5</td>
</tr>
<tr>
<td>A Letter to Parents about the “New Mathematics”</td>
<td>7</td>
</tr>
<tr>
<td>Sister Mary Petronia, S.S.N.D</td>
<td></td>
</tr>
<tr>
<td>A Letter to Parents of Elementary School Students</td>
<td>12</td>
</tr>
<tr>
<td>Rosemary Reuille Irons</td>
<td></td>
</tr>
<tr>
<td>Mathematics Work Stations: Hands-On Learning to Deepen Understanding:</td>
<td>19</td>
</tr>
<tr>
<td>Functions and Geometry</td>
<td></td>
</tr>
<tr>
<td>Tim Deis and Jodean Grunow</td>
<td></td>
</tr>
<tr>
<td>Engaging Our Students in Upper-Level Mathematics: Trigonometry Work</td>
<td>24</td>
</tr>
<tr>
<td>Stations</td>
<td></td>
</tr>
<tr>
<td>Dave Ebert</td>
<td></td>
</tr>
<tr>
<td>Barbie vs. The World: A Social Justice Lesson</td>
<td>26</td>
</tr>
<tr>
<td>Amy Young</td>
<td></td>
</tr>
<tr>
<td>The Wisconsin Association of Mathematics Teacher Educators: Supporting</td>
<td>29</td>
</tr>
<tr>
<td>Teacher Education and Professional Development in the Badger State</td>
<td></td>
</tr>
<tr>
<td>Michael D. Steele</td>
<td></td>
</tr>
<tr>
<td>WMC Puzzle Page</td>
<td>32</td>
</tr>
</tbody>
</table>
We are excited to bring you the 2017 Special Issue of the *Wisconsin Teacher of Mathematics*. This issue is divided into two sections. The first offers a celebration of an influential figure in the history of mathematics education in Wisconsin, Sister Mary Petronia Van Straten. As you will read in this issue, she was an outstanding educator and leader in the field with a great dedication to improving the teaching and learning of mathematics. Her influence helped to shape the Mathematics Department at Mount Mary College, the Wisconsin Mathematics Council, the Wisconsin Teacher of Mathematics, and mathematics education within the United States. In 1965 Sister Petronia’s article “A Letter to Parents About the ‘New Mathematics’” was published in the *Wisconsin Teacher of Mathematics* and later reprinted in NCTM’s *Arithmetic Teacher*. We have reprinted her article (which is now over 50 years old) because it provides much food for thought about how mathematics has (and has not) changed.

One of Sister Petronia’s students, Rosemary Reuille Irons, has prepared “A Letter to Parents of Elementary School Students,” which offers updated suggestions for parents. Irons has also prepared a brief biography of Sister Petronia and provided comments from many former students. Sister Petronia’s influence continues with an annual scholarship in her name that is awarded to a preservice teacher.

The second part of issue touches on a range of different topics. Tim Deis and Jodean Grunow’s article is the second in a series that focuses on work stations. In this installment, the authors discuss how to create work stations to explore a wide range of topics related to functions and geometry at the high school level. Complementing Deis and Grunow’s article, Dave Ebert continues his series “Engaging our Students in Upper-Level Mathematics” with ideas for creating work stations related to trigonometry. Amy Young presents a statistical investigation that is a rich example of how social justice can be brought into the mathematics classroom. In “Barbie vs. The World: A Social Justice Lesson,” Young considers how children’s toys might provide data for investigating society’s perception of body image.

The final piece by Michael Steele is the announcement of a new organization, a Wisconsin affiliate of the Association of Mathematics Teacher Educators (WI-AMTE). Steele shares some of the history of the larger organizations and outlines the many opportunities that WI-AMTE will provide to the many people involved in mathematics teacher education within our state.

As always, we encourage readers to share the great things happening in their classrooms! We hope that you enjoy this issue and find the articles to be useful in your classroom practice. If you have an idea for an article or questions about submission, please contact us.

Joshua Hertel
Jennifer Kosiak
Jenni McCool
The Gift of Giving in Mathematics Education

By Jennifer Kosiak, WMC President

Maya Angelou once said, “When you get, give. When you learn, teach.” As an educator, I have consistently embraced the first part of this quote in my daily and professional practices. However, the latter part of this quote made me wonder about how I give back to my community, including the mathematics education community.

As I reflect on the entirety of this quote, many Wisconsin educators come to mind. This special edition of the Wisconsin Mathematics Teacher journal is dedicated to Sister Mary Petronia Van Straten, one of those influential educators who both taught and gave to our mathematics education community. You may have heard her name in the context of a scholarship for a preservice teacher at a Celebrate Wisconsin event held at the WMC Annual Conference. If so, you may be like me and have only heard part of her teaching and giving career. So, for this special edition, I decided to do a little research in the archives of WMC documents. Here is what I found!

The WMC newsletter was first published in fall 1972 and it is not surprising that in this inaugural edition, Sister Mary Petronia Van Straten was elected to the WMC Board as the Treasurer (see Figure 1).

For many years, she gave back to the mathematics community as the treasurer and president of WMC. But on further investigation, her teaching and giving career began much earlier. In 1983, Sister Mary Petronia Van Straten received the first WMC Distinguished Mathematics Educator Award. Her biography from the WMC Newsletter is shown in Figure 2.

As seen in her biography, part of her teaching and giving career was related to presentations and publications centered on student learning. For example, she published the problem solving task shown in Figure 3 in the fall edition of the 1973 WMC Newsletter.

This special issue of the WMC Mathematics Teacher Journal is focused on her most famous 1965 publication in the WMC Wisconsin Mathematics Teacher journal that was also reprinted in 1966 in the National Council of Teachers of Mathematics journal, the Arithmetic Teacher (now called Teaching Children Mathematics). In this article, she outlined the what and why of the “new mathematics” to parents. The central focus of this article is still relevant today!

Figure 1. Excerpt from Volume I, Issue I of the WMC Newsletter.
This relevance is evident in my favorite quote from the article: “The modern program regards mathematics as a system of thinking rather than a set of arbitrary rules, as a system better learned by understanding of structure and principle, than by mere memorization of facts.” Take a moment to reflect on this quote and its relationship to the current Standards for Mathematical Practice (SMP) in the Common Core State Standards for Mathematics, which was published some 45 years later.

Do you see SMP 2, Reason Abstractly and Quantitatively, which asks students to “make sense of quantities and their relationships in problem situations” (p. 6)? This SMP is highlighted in the article by the story situation: “Mary had 7 jacks. Sue gave her some more jacks. Now Mary has 10 jacks. How many jacks did Sue give her?” In the article, Sister Mary Petronia Van Stranten emphasized that students should be encouraged to represent story situations and connect these representations to the arithmetic sentence needed to solve a given task.

Do you see SMP 7, Look for and Make Use of Structure, or SMP 8, Look for and Express Regularity in Repeated Reasoning, which call for students to “discern a pattern or structure” in order to “look both for general methods and for shortcuts” (p. 8)? Both these SMPs are evident...
in her discussion of the scaffolding method for long division as repeatedly “removing a number of sets at one time.”

The examples above highlight only a small set of the gifts that Sister Mary Petronia Van Straten gave to the mathematical community throughout her long career. This led me to my final research finding from the WMC archives. In the 1987 spring issue of the WMC Newsletter, we find the sad announcement of her passing. As shown in Figure 4, we find that this announcement was also a celebration of her teaching and giving career.

But there is more to the story. In January 1988, a WMC preservice teacher education scholarship was renamed in honor of the work of Sister Mary Petronia Van Straten. The original scholarship was $700 and was awarded to Ms. Jennifer Young of Marquette University at the 1988 Annual Conference. Today, this gift of giving is still part of the Annual Conference award ceremony at the Celebrate WMC Thursday evening event. This scholarship, along with the Ethel A. Niejahr and Arne Engebretsen scholarships continue to be part of WMC through our Wisconsin Mathematics Education Foundation (WMEF). Information on these scholarships can be found at http://wmefonline.org.

Now back to the original quote from Maya Angelou: “When you get, give. When you learn, teach.” Please reflect on how you embrace the gift of giving in your own classroom or in the larger mathematics education community. One way that I plan to embrace this quote is by donating to the WMC foundation, WMEF, to support teachers and students across the state of Wisconsin.
Sister Mary Petronia Van Straten: A Short Biography

Sister Mary Petronia Van Straten, School Sister of Notre Dame, was a mathematics teacher and chair of the Mathematics Department at Mount Mary College, now Mount Mary University, from 1947 to 1987. She completed her doctorate at Notre Dame University. Sister Petronia was past president of the Wisconsin Mathematics Council and was instrumental in the development of mathematics education in the State of Wisconsin. Her leadership talents guided many other university and college educators and students throughout her career.

Sister Petronia’s professional record includes numerous articles, offices held, lectures given, and awards received at the national, state, and local levels. In 1983, Sister Petronia was awarded the Distinguished Mathematics Educator Award from the Wisconsin State Department of Public Instruction. She also received the Outstanding Educator of America award, received a citation from the Wisconsin Mathematics Council in 1985, and was included in the Who’s Who of American Women.

As a mathematics education leader, Sister Petronia had much influence on many teachers and individuals. She was an outstanding “teacher of teachers.” Her respect for individual learners and influence is still felt around the world years after her death in 1987. She had a special gift for taking abstract subject matter and finding ways to make it concrete—taking the complex and breaking it down to its simplest parts. Rather than teaching by telling, she would formulate question after question to draw out the answers so that students discovered solutions on their own. She helped all of her students by sharing her enthusiasm for mathematics and by modeling enjoyment of the experience of learning, especially learning mathematics.

Sister Petronia loved, cared for, and understood each individual student through her nurturing teaching approach. She has had a continued influence on mathematics education worldwide. Many former students commented on Sister Petronia’s influence in their teaching and careers. Some of these comments are included here.

While I was at MMC, Sister Petronia was chairman of the math department, my major professor, boss, teaching mentor, and friend. Her enthusiasm for mathematics was contagious at both elementary and higher levels. She was an effective spokesperson for the discipline, traveling to many venues to give presentations and speeches. Her example inspired my study of mathematics and the pursuit of a teaching profession in that field. (Karen Johnson Clark, Class of 1968)

As a math major at MMC, I appreciated Sr. Petronia’s patience and care in answering questions and always making sure her students understood the concepts she was teaching. Her influence made me a better teacher, especially when I was teaching college students who were underprepared in math, had math anxiety, and/or had bad experiences in math classes in their earlier years. Sr. Petronia was an outstanding example of an outstanding teacher. (Marilyn [Vanden Burgt] Ebben, Class of 1963)

I had always been drawn to the subject of mathematics but it only took one semester with Sister Petronia, and I was ready to commit to having mathematics as my major. She presented math in a way that was so simple, yet elegant. I absolutely loved the way she could graph on the chalkboard—her circles and

This short biography is an adaptation of articles on Sister Petronia’s career written by Rosemary Reuille Irons.
parabolas were almost perfect and she did them freehand! I was encouraged to become a secondary math teacher—something I thought was impossible since I hated getting up in front of a group and speaking. Somehow she made me forget that “imperfection” and just concentrate on learning math and learning how to present math to a class. My first job was teaching six classes of Algebra 1 to freshmen at Benilde High School in Minneapolis. It happened to be an all-boys high school and I was the only female teacher. I know that Sister Petronia was instrumental in getting me that job with her wonderful letter of introduction and phone calls. I also think it helped that this is a Catholic High School. I received most all of my mathematics instruction from Sister Petronia and thus find that my approach to teaching mathematics followed her lead. I always gave my students a Letter of Introduction and then asked each student to write me back a Letter of Introduction about themselves, so I would get to know them better. In that letter, I wrote how much she influenced my choice of profession and how she made me realize that students need to feel successful and encouraged. She instilled in me a genuine love of number and patterns and that is what I wanted to do for my students everyday in the classroom. Sister Petronia was a part of my teaching every day during the 25+ years I taught secondary mathematics. (Delores [Lori] Ziemann Peirick, Class of 1968)

Sister Petronia was the best teacher I ever had, bar none. I tried to model my own teaching after her methods. She was kind, always well-prepared, well-organized, and very thorough. I was a math major and a physics minor so I did not have any particular math anxiety. However, when Sister Petronia taught Elementary Math for Elementary teachers, she helped to dispel much of the math anxiety in the Elementary Ed students. (Therese G. Smith, Class of 1963)

Sister Petronia was my teacher and mentor. She organized classes and topics so five of us could major in mathematics and elementary education. Her leadership example made mathematics relate to the real world. She emphasized the importance of the beginning mathematical concepts and each child’s need to have a picture of the ideas. As a result of her influence, I have been able to develop my career in early childhood mathematics education creating interesting resources for early childhood mathematics. From teaching in elementary schools to university lecturing, I have had a teaching career trying to emulate her influence of enthusiasm, enjoyment and love of teaching young children. Her approach is with me now as I work with early childhood teachers in Australia, the US, and Thailand. Her influence on my teaching and me personally has been career forming. I am grateful for her devotion to students and her teaching on my life. (Rosemary Reuille Irons, Class of 1968)

To honor Sister Petronia’s mathematics education leadership, the Wisconsin Mathematics Council has a yearly scholarship in her name to carry on her influence today in our mathematics education teaching lives.
A Letter to Parents About the “New Mathematics”

Sister Mary Petronia, S.S.N.D, Mount Mary College, Milwaukee, Wisconsin

Dear Parents:

I understand that you are anxious about your children and about what is going on in mathematics today. You are wondering what this “new math” is all about, whether it is better than the way you learned it, whether it is here to stay, and more. That is precisely why I am writing this letter to you, hoping I will answer some of these questions for you.

Let me first say a word about the “why” of the changes. Perhaps the most important reasons for the changes are due to the tremendous advances that have been made in mathematical research, the automation revolution, and computing machines. Not only can machines carry out calculations much more quickly than man; they can also perform computations that were formerly completely impossible. I might cite another reason for the changes—namely, that the traditional program was not sufficient to train persons for our rapidly changing space age. We cannot predict what skills those who are in our schools today will need in the future. This, indeed, should make us emphasize the student's need for discovery and understanding, so that he will be alert and resourceful in applying mathematics to a variety of situations once he is out of school. Scientific and technological advances have demanded changes in mathematics, and mathematics has become “alive” because of these changes.

You may wonder who really started these changes. Individuals and groups had been experimenting with certain phases of mathematics at various levels in different places in the early 1950’s. The UICSM (University of Illinois Committee on School Mathematics), one such group, worked on developing a new curriculum in mathematics for secondary schools. But it was the College Entrance Examination Board's Commission on Mathematics that really started national reforms, about 1959. The School Mathematics Study Group, generally referred to as SMSG, took up where the Commission left off. Many other groups, like the Greater Cleveland Mathematics Program, began to plan and develop programs that would be both mathematically correct and pedagogically sound. Currently, changes in mathematics are so extensive, so profound, and so far-reaching in implication that they can only be described as a revolution.

You ask me to tell you what “modern mathematics” is! Lest my letter become a dissertation, let me try to explain this rather briefly. To my way of thinking, “modern mathematics” is as much a point of view as it is a body of materials; and consequently, it might be wiser to speak of a modern program of mathematics. The modern program involves a change in both emphasis and content. Some new topics have been added, it is true; but there is also a decided difference in attitude, purpose, and approach. The modern program regards mathematics as a system of thinking rather than a set of arbitrary rules, as a system better learned by understanding of structure and principles than by mere memorization of facts. Essentially, then, it implies a reorganization of traditional subject matter, some new content, and a decidedly new teaching approach, all within the framework of the unifying concepts that form the basis of the structure of mathematics as a whole. Its main objective and primary concern is to have the student understand why he does what he does. Its approach is one of investigation and discovery. It aims to make mathematics intellectually exciting and challenging.

Many programs and projects in mathematics have come into being but the core of all of them is the belief that students must understand why they are doing what they are doing. The thought behind them, I think, is that mathematics can be more effectively learned and retained if the approach is one of investigation and discovery of the structure and principles of mathematics. Permit me to quote from the pamphlet, “The New Mathematics Curricula—What and Why,” published by the National Council of Teachers of Mathematics: “It is true that there are many curricula, but it is highly significant that the fundamental philosophies of them all are in very close
I feel certain you would be disappointed if I did not attempt to tell you something about topics your children are bound to be discussing. Without doubt, one of these is the topic of sets, which is one of the basic building blocks of mathematics and permeates all branches of it. A set may be thought about as a collection of objects. Each object that belongs to a set is called an element or member of the set. Each set must have a definite defining property, a means by which one can tell when an object does or does not belong to the set. For instance, I might speak of the set of states in the United States. Then Wisconsin would be a member of this set, whereas Holland would not. Sets may be designated in a number of ways. At a very elementary level, one might draw pictures of the objects that belong to the set and have them enclosed by some type of frame, as in the diagram below.

At higher levels one would use some capital letter to name the set and enclose the members of the set between braces, separating them with commas. For example, for $A = \{\text{Wisconsin, Wyoming, Washington, West Virginia}\}$, read, “$A$ is the set whose members are Wisconsin, Wyoming, Washington, and West Virginia.”

Would you care to increase your vocabulary further? Then consider this idea. If two sets can be matched element for element, then we say the sets are equivalent or can be put into a one-to-one correspondence. This is not a difficult concept to grasp. Think about the set of stars in the American flag. Why did we, not too long ago, change the flag so that it would have fifty stars? I like to think it was done simply to preserve the one-to-one correspondence between the set of states in the United States and the set of stars in the American flag. You would agree, would you not? You may not be aware of this, but actually, when your child really learns to count, he is simply setting up a one-to-one correspondence between a set of objects and a set of number names. Right?

Now, if you are still with me, consider a set of three chairs, a set of three apples, a set of three pencils, and so on. You cannot help but notice, can you, that each of these sets has some property in common. Each one shows “threeness.” So we say that each of these sets, as well as any set equivalent to any one of them, has the cardinal number three. In this way I am sure you can see that any set you can conceive of has some number to correspond with it. So number is a property of every set. A number, then, is an idea, an abstraction. So, in order to talk about it, we need a number word and a number symbol. Numerals are names for numbers or symbols used to represent numbers. We who teach mathematics think it is important that a student know the difference between a number and a numeral, so that he may distinguish between actually operating with numbers and merely manipulating symbols for those numbers. In essence it is simply distinguishing between a thing and the name of a thing.

I am certain it would be easy for you to see that a particular number may have many names. For example, the number five may be named by any of these symbols: $3 + 2; 8 - 3; 1 \times 5; 1 + 2 + 2; \sqrt{25}$. This idea of finding different names for numbers opens a vast and exciting field for children to explore. Actually, much of mathematics is finding different names for numbers and then being creative enough to choose that name which is most convenient for a particular situation.

This reminds me of an incident I had the joy of experiencing in a fourth-grade classroom. Some lad had this subtraction to do:

\[
\begin{array}{c}
5000 \\
-2765 \\
\hline
\end{array}
\]

Remember how you learned to do it? Probably it went like this: “5 from 0, I cannot take, so I borrow a ten; but there are no tens, so I borrow a hundred,” and so on. This is not the approach this lad took. No, he knew that 5000 had many names, and he chose the name 500 tens. So his reasoning went like this: “I cannot take 5 ones from 0 ones, so I will take a ten. I have 500 tens. If I take one ten and change it to ones, then I will have 499 tens and ten ones.” Then he was
all set to subtract. You cannot help but agree that this lad's method was efficient and that his performance gave evidence of understanding.

I was talking to you about sets. Do you mind if I pursue this topic a little further? I would like to show you how the properties of operations on numbers can be derived from them. Suppose we think about a set of three girls and then imagine that a set of two girls joins it. We have a new third set whose number is five. The mathematical sentence that fits this situation is: $3 + 2 = 5$. If we had the set of three join the set of two, again we would come up with a set of five, but the mathematical sentence would be: $2 + 3 = 5$. These situations and many more like them would help children discover that it does not make any difference in what order numbers are added—in other words, the commutative principle. No doubt you learned this too when you went to school; but in all probability you learned it as a rule, without having the opportunity or fun of discovering it for yourself. How I would like to tell you about those other principles, the associative and the distributive principle! But that will have to wait.

I am quite certain that you have been hearing your children speak about number bases, as it is a topic most children find very fascinating. What's that? you ask. The base of our numeration system is ten, which means we need ten basic symbols and we group by tens. Also, it means that whenever we write a numeral, each digit in the numeral has a certain value, which is determined by multiplying the number the digit represents by the number associated with the place the digit occupies. Let me illustrate it this way.

If, then, we decided to change the base to some number other than ten, what would be the consequence? Simply this: the number of basic symbols needed would change, the grouping would differ, and the names of the places in a numeral would change. Suppose I had as many sticks as are pictured in the frame below:

Suppose, further, that I were going to count the number of sticks. In base ten, I would group by tens as follows:

```
1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1
1
```

So I would represent the number of sticks as 21, meaning two sets with ten in each set and one remaining. Were I counting the same number of sticks in base nine, I would pick up one group of nine sticks, then another, and notice there were three remaining. Picturing this situation, I would have:

```
1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1
1 1 1
```

Consequently I would represent the number by writing $23_{\text{nine}}$ meaning two sets with nine in each set and three remaining, and read “two, three, base nine.” If I cared to do so, I could continue representing the grouping and naming the same number in other bases. For example, in base seven:

```
1 1 1 1 1 1 1
```

I would record the result as $30_{\text{seven}}$ meaning three sets with seven in each set and no more, and read “three, zero, base seven.” In base five:

```
1 1 1 1 1
```

So I would record this as $41_{\text{five}}$, meaning four sets with five in each set, and read “four, one, base five.”

I surmise that you have heard about the so-called “new” division. Actually it is not new, as it was used years ago. What it amounts to is a method of recording division, a method that lends itself well to helping a child understand what he is doing. Not only that, but it also permits each child to work at his own level of performance. Let me illustrate with this division:

```
35 ) 1507
```

This means I have 1507 things and I want to see how many times I can remove a set of 35 things. Naturally I could find out by removing one set at a time, which would surely be a slow and tedious process. However, I could speed up the process by removing a number of sets at one time and recording how many I remove each time in a column at the right side of the example. Using this procedure, I could perform the above division in any of the following three ways, as well as in many other ways.
Besides making division understandable to a child, this method has another benefit. A child's own laziness will be a spur to urge him to improve his performance. What more could one desire?

Another innovation in a modern program of mathematics is the early use of equations. Children are encouraged to think of problems as “stories,” visualize the action going on in the story, translate this action into an arithmetic sentence, and then perform the necessary computation to find the answer. For example:

Mary had 7 jacks. Sue gave her some more jacks. Now Mary has 10 jacks. How many jacks did Sue give her?

The arithmetic sentence would be: \(7 + \square = 10\). Many children would readily know that the \(\square\) should stand for the number three. If a child did not, then one would picture the situation as below to help him see that the necessary computation is subtraction.

\[
\begin{array}{c|c}
\hline
35 & 1507 \\
350 & 10 \\
1157 & 807 \\
350 & 10 \\
807 & 107 \\
350 & 10 \\
457 & 105 \\
350 & 10 \\
107 & 3 \\
\hline
& 2 \\
& 43 \\
\hline
\end{array}
\]

The picture shows the 10 jacks that Mary had after Sue gave her some. The jacks circled show the set Mary had to begin with. Then I think it would be evident that this set would have to be removed from the set of ten to find out how many jacks Sue gave Mary.

I know you are wondering whether children must still learn their “times tables” in a modern mathematics program, since this is a question I have been asked frequently. The answer to that question is definitely yes. Yet, I would hope, in the light of a modern program, that the teacher would have the children understand how to find the answers to the facts in the first place, and likewise would use interesting methods of having children endeavor to attain automatic responses to these facts, rather than resort to pure drill. Many educators do not like the word “drill,” so perhaps they would prefer we call it “sustained attack.” Call it by what name you will, it is my opinion that even talented children need a certain amount of drill to fix ideas and make them their own.

I understand you are lamenting the fact that you can no longer help your children with their mathematics. But is this really the case? I suppose it depends, in part, on what you mean by help. If you mean just telling your children what to do or, worse yet, doing it for them, then I suppose you cannot, because of the transition going on in mathematics. But, truly, this would not be helping but hindering them; for they must understand why they do what they do. Perhaps we should make a distinction between concepts and skills. Certainly, as far as concepts are concerned, it is the teacher’s duty to see to it that the children he teaches understand them. So, in this respect, you might let your children tell you what they are doing in mathematics. If they really know and understand what they are doing, then they should be able to make it clear to you. If they do not, you might suggest they ask their teachers for more explanation. In this way you yourself can learn enough about “modern mathematics” to appreciate that your children are learning with understanding and because of that their learning should be much more lasting. However, when it comes to skills, I do think you can help. To take a specific example: you know there are sets of cards one can procure that contain the basic facts in the different processes: addition, subtraction, multiplication, and division. After your child has learned the facts with understanding, then he must work toward and attain the goal of automatic response. You could be instrumental in helping him reach that desired goal by listening regularly as he goes through the cards, reciting the answers to the facts. Perhaps you would even care to time him, keep a record, and in that way urge him to improve his skill. There is not time in a regular school day for the necessary drill the majority of children need, and any classroom teacher would appreciate your help in this regard. Also, you could see to it that your child gets practice in computation, so that he may continuously make progress in accuracy and speed.

I sincerely believe that if “modern mathematics” is taught properly, it is decidedly better than the way
you and I learned it. Not only that, but it should produce much better thinkers. So I would suggest that the next time you count your blessings, you see to it that “modern mathematics” is on your list.

In the hope that you would be interested in learning more about “modern mathematics,” I am suggesting a list of the names of pamphlets and books that were written especially for parents.

By your own active interest, positive attitude, and steady encouragement, you can make an exceedingly important contribution to your children’s success in a modern program of mathematics.

God bless you and all of us teachers and help us to give our best efforts to teaching and training our children.

Sister Mary Petronia, S.S.N.D.
A Letter to Parents of Elementary School Students

Rosemary Reuille Irons

In 1966, Sister Mary Petronia, School Sister of Notre Dame, wrote a letter to parents about mathematics education during a time of major changes in the teaching of elementary school mathematics. At that time, set theory, other bases, and long division were included in the curriculum. Sister Petronia wanted to support parents in understanding some of the changes in the approach to teaching mathematics and utilize research knowledge of children learning mathematics and the effects of technology. Sister Petronia was my mentor, and I would like to continue her work in supporting parents and caregivers. This letter to parents reflects the current changes in focus for the teaching of mathematics in the elementary school and describes how you can help.

Dear Parents,

Mathematics is alive in the world. You use it every day in all aspects of life whether buying gas or groceries, reading a map, using a satellite navigation device in the car, determining what bus to take, solving a puzzle, or using a calculator or cell phone. Mathematics is what makes things in the world work. Mobile phones, security systems, cash registers, internet banking, and digital appliances all have mathematics as the basis of their electronic core (Figure 1).

If your child is a student in the years Kindergarten to Grade 2:

You are your child’s first teacher. He or she listens intently to you and models your actions and words.

Your child sees your interests and involvement in home, leisure, and work life. When your child is young, modeling new words, pointing out interesting things in the environment, encouraging observation, and sparking motivation to learn new information is part of your interaction on a day-to-day basis.

In school, your child will be using language of specific mathematics words as he or she works with problem solving in learning experiences. Hearing stories with mathematical concepts and acting out these stories helps your child learn new mathematical terms (Figures 2 and 3). Activities to act out stories using mathematical language are part of learning experiences in school.
Figure 2. Children represent stories with materials and symbols.

Figure 3. Children match a total on the balance to equal the red and yellow counters. This balancing idea supports the understanding of equals before the symbol is introduced.

The concept of subtraction has varied terms. The chart in Figure 4 shows some of the mathematical and real-world language that reflects the subtraction ideas. Problem solving stories, which are usually given verbally or in written form, never contain operation signs. Your child determines the operation needed to solve the problem by interpreting the words of the story situation.

Formal mathematics that is symbolic, such as $8 + \_ = 12$, is also represented with stories. For example, “We put 8 eggs in the carton, but it isn’t full. How many more do we need to fill the carton?” Another example is presented in Figure 5.

Figure 4. A chart showing real-world language related to subtraction.

Figure 5. A mathematics story.
Today, reasoning and thinking are the focus when your child encounters mathematical activities in school. However, there are still some basic facts and knowledge of number and geometry that are essential. Success in learning new ideas depends on a good foundation in knowledge of number. Number has many facets that your child will learn. Ideas about number (Figure 6) are presented as quantity (5 happy faces), relative position (knowing numbers through relationships to other numbers such as 5 is one more than 4), ordinal (5 is the fifth counting number), and as a label (Bus 5 or a 5 on a mailbox).

Building on this number knowledge as well as knowledge of addition and subtraction concepts, your child will memorize and learn strategies for number facts (what you may have learned as the tables). Knowing the number facts is taught through a strategies’ approach, and the facts are required for later mental computation work in school and life. A major focus of elementary school mathematics is learning strategies for mental computation. One strategy, using knowledge of doubling numbers to find the sum of two numbers, is presented in Figure 7.

Your child will be asked to use his or her mind first and then use a calculator if needed to figure out a problem. In school mathematics, children write as a way to keep track of their thinking and to help them solve a problem.

Calculators and computers have not removed the need to learn mathematics. The emphasis in the mathematics that your child needs to learn has changed because of the advances in technology. Your child will not be working on pages of symbolic problems because technology can perform those calculations. Automatically working out procedures, which you may have learned to answer symbolically represented problems, does not help your child to think about the problem. Concepts such as addition, subtraction, multiplication, and division are carefully taught to ensure that your child has a picture of the idea for each operation. Being able to visualize the operation, and its relationship to the reverse operation, builds a basis for problem solving. Understanding operation concepts,
This is often used in statistics and probability.

Serious Sundaes

<table>
<thead>
<tr>
<th>Flavours</th>
<th>Toppings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vanilla</td>
<td>Toffee</td>
</tr>
<tr>
<td>Strawberry</td>
<td>Fudge</td>
</tr>
<tr>
<td>Chocolate</td>
<td>Caramel</td>
</tr>
<tr>
<td>Berry</td>
<td></td>
</tr>
</tbody>
</table>

How many different combinations can you create? You can have one flavour and one topping.

The combinations model

The set model

3 bags of 4

3 stacks of 4

The length model

3 trains of 4 cars

3 jumps of 4

The array model

This model helps develop number facts

4 by 6

6 by 4

Figure 9. Models for multiplication.

If your child is a student in the years Grade 3 to 5:

For Grades 3 to 5, the most important mathematical topics are a thorough understanding of the concepts of multiplication and division, memorization of multiplication number facts, the relationship of multiplication and division, and fraction knowledge. Your child will be learning the various models for multiplication shown in Figure 9: the set (group), length–number line, array, and combination.

Your child is familiar with division long before they work with the division symbols. He or she is able to share quantities that work with making equal groups. Unlike other operations, your child uses the term divide in their everyday language. The two types of division—sharing (partition) and repeated subtraction (quotition)—are studied and represented using equal groups and array models (Figure 10). These models help to show how division can be linked back to multiplication.

Create an everyday situation that forces the movement of the counters.

“The three groups of four is twelve.”

The reverse process will help make the link to division.

“The twelve put into three groups is four.”

Figure 10. Models of division.
There are specific number fact strategies that your child’s teacher can explain to you, or you can have your child describe the strategies he or she is learning. Your role is to help your child practice and memorize the multiplication number facts so they can recall them instantly. Knowing the number facts is important for knowledge of both common fractions and decimal fractions. The area and length models are used to review unit fractions (one part of the equal parts of one whole). Figure 12 shows one sixth using an area model and three eighths on a number line model.

Your child writes fraction symbols and words to match shaded regions and lengths and vice versa. He or she learns the names for the numbers in the fraction that describe the number of equal parts in the whole (denominator) and the number of parts that have been shaded (numerator). The number line is closely related to the length model, but there are important distinctions between the two representations. The number line model emphasizes the aspect of a number as a distance from zero, helps to develop the idea of equivalent fractions, and links common and decimal fractions.

Decimals are represented pictorially before dealing with those numbers as symbols. Students in Grade

Figure 12. Representations of one sixth and three eighths.

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Decimals are represented pictorially before dealing with those numbers as symbols. Students in Grade
4 should have a good picture of decimals in their minds. Figure 13 shows the different model pictures representing one tenth.

Decimal notation, for a quantity, indicates where the “ones” are to help represent the whole ones and the decimal fraction parts. For example, four wholes and six parts can be represented using an area model (Figure 14) or a number expander (Figure 15). The purpose of the decimal point is the most important thing to remember!

Learning about the hundredths follows the same sequence using a picture of the one square divided into a hundred squares. The tenths and hundredths can be represented and visualized in the one square (Figure 16).

If students have a good understanding of these basic number and operations concepts, they are more confident when they work in problem solving situations. Your child will be using his or her knowledge of number and applying that knowledge to measurement and data problem situations. The world is filled with geometrical ideas, so topics in geometry are much more important today. The focus in the geometry strand is location in space, characteristics of three-dimensional objects, features of two-dimensional shapes, and identification of angles. Ask your child’s teacher to describe the geometrical aspects developed and reviewed at their grade level. Mathematics is a language and your child will be converting words from problems and investigating mathematical symbols to find results. Problem solving requires the interpretation of words and experimenting to find reasonable answers.

I hope that the descriptions of some of the mathematical ideas that your child will encounter are helpful. Remember to listen to your child and have him
or her tell you what they are learning about mathematics at school. The goal of mathematics is reasoning. Success in mathematics depends on basic ideas and the building of new concepts on established concepts and skills. Your child’s enjoyment of learning about mathematics is supported by you. Encouraging them to “give it a go” or “keep on trying” develops the goals and skills to be a lifelong learner.

Rosemary Reuille Irons
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Figure 16. Representation of 0.35.

Are you looking for an opportunity to write for a professional journal?

The Wisconsin Teacher of Mathematics invites you to share what you’re doing in your classroom, research with which you may be involved or a great tip you think other educators could use. Each issue offers articles for the different grade bands so we’re looking for submissions focusing on PK through post secondary education.

Submissions are welcome. If you have questions or wish to submit an article for review, please email Josh Hertel (jhertel@uw lax.edu) or visit the WMC website for more information (http://www.wismath.org/Write-for-our-Journal).

The submission deadline for the Fall 2017 issue is September 1, 2017.
Mathematics Work Stations: Hands-On Learning to Deepen Understanding: Functions and Geometry

Tim Deis and Jodean Grunow, University of Wisconsin-Platteville

This article is the second of a three-part series of articles that shares hands-on activities that have been implemented in teacher workshops. The previous article addressed investigations that explored the subject of numbers and operations. This article will address the subjects of functions and geometry and the activities used to engage students.

As instructors who teach mathematics, the importance of connecting various representations of a topic is central to deep understanding. The National Council of Teachers of Mathematics (2000) Principles and Standards for School Mathematics document states, “The opportunity for students to experience mathematics in a context is important” (p. 65). These investigations allow students to touch and visually explore the applications of various mathematical concepts. These types of representations are very important to a student because they help the student to advance their understanding of the concepts and helps engage them in mathematical discussions (Arcavi, 2003; Stylianou & Silver, 2004).

As a result of numerous teacher workshops (Designing and Using Value Added Assessments, STEM Connects, and I^2 STEM), workshop participants explored and implemented a type of differentiated instruction through the use of centers. A learning center is a classroom area that contains a collection of activities or materials designed to teach, reinforce, or extend a particular skill or concept (Kaplan, Kaplan, Madsen, & Gould, 1980). The value of a center is that it provides the instructor with observable assessment of student work and problem-solving activities that can be experienced in groups. The activities also allow students to learn as they explore through their own experiences.

When would a teacher use mathematics learning centers? If a concept has several facets, each worthy of investigation, the teacher could choose to develop each portion at a center. If a concept is “deep,” various centers could be developed to gradually deepen understanding with each investigation. In that case, the teacher might choose to have a specific rotation. Prior instruction could be reinforced with parallel, connecting, or reflective centers. Because learning centers provide for movement and individual and group problem-solving scenarios, motivating and interjecting them into the curriculum can be nonroutine and stimulating.

There are many resources available for elementary teachers to help create centers in the classroom. Two examples of good resources are Debbie Diller’s Math Work Stations: Independent Learning You Can Count On, K-2 and Marilyn Burns’ About Teaching Mathematics. However, finding a book on creating centers for high school math content is more difficult.

This article will focus on the centers that were developed as a result of the workshops based on conceptual categories from the Common Core State Standards, in particular functions and geometry.

These centers were created for individuals who had a wide range of mathematical experience. They were used to help each individual explore content based on the their level of understanding. The centers were used to explore a wide range of topics within each standard. The activities may not fit the exact definition of a center because they will not address a specific topic or concept within a standard. The following investigations for functions and geometry can be (and were) adapted by instructors for use in their classroom.

Functions

Understanding the relationship between variables and how one variable interacts with another, is critical to the mathematical development of a student. Function serves as the foundation for understanding, designing, and developing technology; predicting events and behaviors; and comprehending financial questions.

The concept of exploring the relationship between two variables is the emphasis of the centers in this standard. Students are asked to explore what happens when one variable from an activity affects another. At each activity, students are required to collect and organize data, graph the information on a coordinate system, and identify the type of function that best models
the experiment. Examples of the function relationships that were explored and adapted are as follows.

1. Coin Toss Function: Participants toss 10 pennies individually onto the lids of a circular Tupperware container. A “bulls-eye” occurs when a penny lands completely in the circular lid. The participants toss the 10 pennies five different times at the following distances: 2 feet, 4 feet, 6 feet, 8 feet, and 10 feet. The numbers of bulls-eyes are recorded. After the data is compiled, it is graphed. The resulting data should best be fit by a linear function.

2. The Candle Function: Participants light a birthday candle and measure the height the candle at 30 second intervals. This continues until the candle burns itself out. After the data is compiled, it is graphed. The resulting data should best be fit by a linear function.

3. The Coffee Pot Function: Participants fill a coffee pot with water. The spigot of the coffee pot is opened for 10 second intervals. At the end of each interval the height of the water in the coffee pot is measured and recorded. This continues until the water reaches the spigot in the coffee pot. After the data is compiled, it is graphed. The resulting data should best be fit by a linear function.

4. The Knot Function: Participants are given a length of rope. The length of the rope is measured and recorded. A knot is tied into the rope and the length is measured and recorded. This process continues until the participant cannot tie another knot in the rope. After the data is compiled, it is graphed. The resulting data should best be fit by a linear function.

5. The Stroop Test Function: Participants will use an experiment from cognitive psychology at this center. There will be lists of color words: red, green, black, and blue. Each list is a different length. Each word will be written in red, green, black, or blue. Two different categories of lists will be used: one on which the color of the ink matches the color word, for example red written in red ink, and a second on which the color of the ink does not match the color word. The first category is called matching and the second category is called nonmatching. Examples of these lists are easily found on the Internet. One individual in the group will be asked to say the color of the ink for each word. The time needed to complete each list will be measured and recorded. After the data is compiled for both categories, each is graphed. The resulting data should best be fit by a linear function but may vary based on individual results.

6. The Bowl Function: Participants are given a kitchen bowl that has a shape resembling a paraboloid (i.e., every vertical cross section of the bowl is a parabola). A half cup of water is placed in the bowl. The participant then records the length of the diameter of the water in the bowl and the depth of the bowl. This data is recorded. Another half cup of water is placed in the bowl, and the length of the diameter and depth of the water is measured and recorded. This process continues until the bowl is full. After the data is compiled, it is graphed with the diameter being the independent variable and the depth of the water the dependent variable. The resulting data should best be fit by a quadratic function.

7. The Cubic Function: Participants are given centimeter paper and are asked to cut out squares that are 20 cm x 20 cm in length and width. Participants cut out congruent squares in each corner of the 20 cm x 20 cm square. The sides of the resulting paper are folded up to form a box without a top. The height and the volume of the box are recorded. The experiment continues with the participant cutting out different sized squares from the corners of the 20 x 20 grid and measuring and recording the height and volume of the resulting box. After the data is compiled, it is graphed. The resulting data should best be fit by a cubic function.

8. The Weather Function: Participants are asked to search on the Internet for the average temperature for their hometown for each month of the year. These temperatures are recorded and graphed. Discussion occurs within the group to determine what function best fits the data. A trigonometric function should best fit the data.

9. The M&M Function: Participants pour a bag of M&M’s onto a paper plate so that the candies
are a layer thick and are spread to the edges of the plate. Participants remove all candies with the M showing on one side. The number of M&M’s removed and the number remaining on the plate are counted and recorded. The M&M’s remaining on the plate are placed into a container and shaken. The M&Ms from the container are then poured back on the plate. Participants remove the candies on which the M is showing. The number of remaining and the number of M&Ms removed are counted and recorded. Continue the process until all the M&M’s are removed. After the data is compiled, it is graphed. The resulting data should best be fit by a decaying exponential function.

The activities in this center were modeled by various mathematical equations. Each activity can be extended in various ways. Participants can be asked to predict the behavior of the activity by using the mathematical model. Different types or sizes of materials can be substituted and explored in each center. For example, two different types of rope can be used in the Rope Function or two different coffee pots in the Coffee Pot Function. Participants can determine how the slopes of lines compare when different materials are used at a center or what an intersection point means in the graphs of the activity being modeled. Many different extensions for each center are possible.

Geometry

The focus of many geometry textbooks is the use of deductive reasoning. This type of understanding is important for individuals in making logical arguments. However, the use of inductive reasoning is also significant. This type of reasoning is used when one makes a conjecture based on the evidence that is available. In fact, many of the theorems in the field of geometry were discovered by using inductive reasoning. If the focus in geometry is just on deductive reasoning, some of the excitement of learning and discovery is lost. The activities in the geometry centers focus on inductive reasoning and having the participant conjecture and discover the relationships that exist.

Examples of the geometry activities that were explored and adapted by participants are as follows.

1. **Triangle Midpoints**: Participants are given a piece of paper and are asked to draw a triangle with all angles acute and all sides a different length. The midpoints of the three sides of each triangle are constructed or found with the use of a ruler. These three points are connected by line segments, and four triangles are formed. Label the inner four triangles and cut them out. This activity can be repeated to get more data. Participants are asked to compare the shape and size of triangles and make a conjecture on a possible relationship. (The four triangles are congruent.)

2. **Perimeters and Similarity**: Participants are asked to draw four triangles on grid paper. Triangle 1 has legs of 6 and 8 units long. Triangle 2 has legs of 9 and 12 units long. Triangle 3 has legs of 5 and 12 units long. Triangle 4 has legs of 10 and 24 units long. The length of the hypotenuse of each triangle is calculated by using the Pythagorean theorem. The ratios of the three sides of each triangle are calculated and compared with the other triangles. The perimeters of the triangles are calculated, and the ratios of the perimeters of each triangle are found. The ratios of the areas of the triangles are also compared with the ratios of the perimeters. Participants are asked to make conjectures based on the data.

3. **Locus Points**: Participants are asked to explore objects in three-dimensional space. The center consists of tennis balls (points), meter sticks (lines), and string. Participants are asked to conjecture on what the following objects would be:

   a. Given a point A, describe the object that is the set of points in a plane that are two feet from the point. (Circle)
   b. Given a point B, describe the object that is the set of points that are two feet from the point. (Sphere)
   c. Given two parallel lines, describe the object that forms the set of points in a plane that are equidistant from the two parallel lines. (Line)
   d. Given two parallel lines, describe the object that forms the set of points that are equidistant from the two parallel lines.
e. Describe the object that is the set of points in a plane that are two feet from a given line. (Parallel lines)
f. Describe the object that is the set of points that are two feet from a given line. (Cylinder)
g. Describe the object that is the set of points in a plane that are equidistant from two given points. (Line)
h. Describe the object that is the set of points that are equidistant from two given points. (Plane)
i. Describe the object that is the set of points in a plane that are equidistant from the sides of a given angle. (Ray)
j. Describe the object that is the set of points that are equidistant from the sides of a given angle. (Half plane)
k. Describe the object that is the set of points in a plane that are equidistant from two intersecting lines.
l. Describe the object that is the set of points that are equidistant from two intersecting lines.

4. **Euler’s Formula:** Participants are asked to conjecture what relationship exists between the number of vertices, number of edges and number of faces of a polyhedron. Models of a tetrahedron, hexahedron (Cube), octahedron, dodecahedron, icosahedron, triangular prism, trapezoidal prism, hexagonal prism, square pyramid, and pentagonal pyramid are used to explore in this center along with a table with the following column headings.

<table>
<thead>
<tr>
<th>Name of polyhedron</th>
<th>Number of Faces (F)</th>
<th>Number of Vertices (V)</th>
<th>Number of Edges (E)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Convex Polygon</td>
<td>Number of Sides</td>
<td>Number of Diagonals from one Vertex</td>
<td>Number of Triangles</td>
</tr>
</tbody>
</table>

From the information collected, participants were asked to conjecture a formula that connects the variables $F$, $V$, and $E$. ($V + F - E = 2$)

5. **Midsegments:** Participants explore triangles and the segments formed from the midpoints of the triangle sides. Participants are asked to create an arbitrary triangle. By construction or the use of a ruler, individuals find the midpoints of two sides of the triangle. These points were labeled $A$ and $B$. The length of segment $AB$ is measured and compared to the length of the side of the triangle parallel to $AB$. This process is repeated for arbitrary obtuse, isosceles, right, and equilateral triangles. Participants are asked to make a conjecture from the relationship that they discover.

6. **Interior Angle Measures:** Participants explore the relationship of the sum of the angles in a convex polygon. They are asked to draw arbitrary convex polygons: quadrilateral, pentagon, hexagon, heptagon, and octagon. For each polygon, a vertex is chosen, and all the diagonals from that vertex to the other vertices of the polygon are drawn. Individuals are asked to fill in a chart with the headings. Participants are asked to fill the last row of the table with an $n$-gon and to conjecture what the sum of the interior angles of this polygon is.

7. **Inscribed angles:** Participants are asked to use a compass to draw a circle on a piece of paper. The center of the circle is labeled $A$. An arbitrary point on the circle is found and labeled $B$. Two chords of the circle are constructed that contain the point $B$. These chords are labeled $BC$ and $BD$. The segments $AC$ and $AD$ are then drawn. Individuals measure the angle $CAD$ and the angle $CBD$. This process is repeated at least three times. Participants are asked to conjecture a relationship that has been discovered.

8. **Chord Angles:** Participants explore the relationships of the measure of angles formed by two intersecting chords of a circle. A circle is drawn, and the center is labeled $A$. Four arbitrary points are found on the circle and are labeled $B,C,D$ and $E$ clockwise about the circle. The intersecting segments $BD$ and $CE$ are drawn with the intersecting point labeled $F$. A compass is used to measure the angle $BFE$. A compass is used again to measure the angle $BAE$ and angle $DAC$ (i.e., the measure of the chords' intercepted arcs). This process is repeated for at least three more arbitrary circles. Participants are asked to conjecture what the relationship that
occurs between the measures of the angle $BFE$ and the measures of the angle $BAC$ and angle $DAE$. ($m\angle BFE = \frac{1}{2} (m\angle BAE + m\angle DAC)$)

This center can be extended to include the angles formed by secant lines that intersect outside the interior of the circle.

9. Chord Measures: Participants explore the relationships of the lengths of the segments formed by two intersecting chords of a circle. A circle is drawn, and the center is labeled A. Four arbitrary points are found on the circle and are labeled B, C, D, and E clockwise about the circle. The intersecting segments BD and CE are drawn with the intersecting point labeled F. A ruler is used to measure the length of the segments $BF$, $CF$, $DF$, and $EF$. The hint is given to look at the product of $BF$ and $DF$ and the product of $CF$ and $EF$. This process is repeated for at least three more arbitrary circles. Participants are asked to conjecture what the relationship that occurs between the lengths of the sides of $BF$, $CF$, $DF$, and $EF$. ($BF \times DF = CF \times EF$).

This center can be extended to include the segments formed by secant lines that intersect outside the interior of the circle.

References and Resources for the Creation of the Center Activities


Engaging Our Students in Upper-Level Mathematics: Trigonometry Work Stations

By Dave Ebert, Oregon High School

In the Spring 2016 issue of the Wisconsin Teacher of Mathematics, Tim Deis and Jodean Grunow shared a wonderful article on using work stations, or learning centers, in the high school mathematics classroom. For years, elementary school teachers have used stations as a teaching tool in their classrooms. Stations allow students to work together in a small group to engage in challenging problems while also differentiating the tasks to meet the varied academic levels and interests of all students. Although there are many resources available to assist teachers in developing stations for use in their classes, these resources are lacking for high school teachers, particularly teachers of upper level high school mathematics students.

The need to create engaging, student-centered tasks for use with our students is well documented. Jo Boaler, in her book Mathematical Mindsets, states that “math facts by themselves are a small part of mathematics, and they are best learned through the use of numbers in different ways and situations” (2016, p. 38). She advocates for the use of games and engaging tasks and calls for us to move from math facts to math excitement. In Principles to Actions, it states: “We need to take action to create classrooms and learning environments where students are actively engaged” (National Council of Teachers of Mathematics, 2014, p. 109).

The need for our students to practice trigonometry is obvious to teachers of precalculus and calculus. The Common Core State Standards for Mathematics include Trigonometric Functions as a domain and “Extend the domain of trigonometric functions using the unit circle” as a cluster heading (National Governors Association for Best Practices & Council of Chief State School Officers, 2010, p. 71). Students must know the values on the unit circle, understand the meaning of those values, and apply those values when solving problems or modeling a situation. Students often struggle with the unit circle and resort to memorization or using tricks to fill out the values of the unit circle. By practicing the values of trigonometric functions and familiarizing themselves with these values in a number of different ways, students can come to know their trigonometric functions and the unit circle.

Work stations are an excellent way to engage our upper level mathematics students while giving them the practice that they need. What follows are three work stations that can be used in an upper level high school mathematics course to engage students in the practice of trigonometric functions. Materials for each of the stations are included in appendices at the end of the article.

Trig War
The majority of high school students are familiar with the card game war. In this game, a deck of cards is shuffled and dealt to two people. The players each turn up one card from their pile at the same time, and the person with the higher card takes both cards. If there is a tie, it is a war. Each player plays three cards face down, and then one card face up. The player with the higher card takes all of the cards.

Trig War uses the same rules as the more familiar game of war, but uses trigonometric playing cards (www.wismath.org/resources/Documents/WTM Issues/TrigWar.pdf). The cards should be copied on cardstock, cut out, and, ideally, laminated. These cards have expressions for sine, cosine, and tangent and include a variety of angle measures from 0° to 360°.

Variations of the game could include cards with angles measured in degrees or radians or could include any combination of the six trigonometric functions: sine, cosine, tangent, cotangent, secant, and cosecant. Students may play with a copy of the unit circle at hand, or they may be expected to know the values of the trigonometric functions. Students should also be encouraged to make sense of the values of the functions. For example, any trigonometric function with a positive value, such as sin 60°, will beat any trigonometric function with a negative value, such as cos 120°.

Trig-Tac-Toe
Although I originally named this game “Trig-Tac-Toe,” my students often call it “Tringo” because
it follows the rules of the familiar game *Bingo*. First, students are given a game board on which they write potential answers in a 3x3 grid. The teacher, or a student volunteer, calls out random trigonometric functions, such as sine 30°. If a student playing the game has the value of that trigonometric function on her game board, she draws an X through that value. The first student with three in a row wins.

There is some strategy involved in setting up the game board. Students familiar with the unit circle know that certain values are more likely than other values and can set up their board accordingly. Variations of the game could include allowing students to use the unit circle, call out angle measures in degrees or radians, or use only the functions sine, cosine, and tangent, which would eliminate some of the numbers given on the game board. In fact, a student volunteer could call out random trigonometric functions by selecting cards from a deck of *Trig War* cards.

**Trigonometry Square Puzzles**

These puzzles originally appeared in an article in the *Mathematics Teacher* but can now be found on the Illuminations website at [http://illuminations.nctm.org/hsactivity/](http://illuminations.nctm.org/hsactivity/). Each puzzle is made up of 16 squares. The sides of each square have an expression. The goal for students is to match up the squares so that equivalent expressions are touching. Figure 1 shows an example of the puzzle.

The square puzzles should be copied on cardstock. The teacher should cut out the individual squares, laminate them if possible, and place the sixteen squares in an envelope. Small groups of students are instructed to match equivalent expressions to create one large 4x4 square, lining up equivalent values.

Oftentimes students need assistance with these puzzles, and different groups of students should be encouraged to share their answers and justify their potential solutions. There are many expressions that potentially match but don’t lead to an overall solution, so students should be warned about this in advance.

**Final Thoughts**

These examples all provide excellent ways for students to practice working with trigonometric functions in an engaging and motivating way. All of these could be used by small groups of students in centers or as a large class activity, and all could be modified based on the individual needs and prior knowledge of the students. Students report a high level of enjoyment when working on these tasks and tend to persevere more than they would on a more traditional task. These activities give students an excellent way to practice their knowledge of trigonometric functions.

**References**


Barbie vs. The World: A Social Justice Lesson

By Amy Young

The ultimate question we are posed with as teachers is: “When are we ever going to use this in real life?” Being realistic, I always respond to my students and admit that they may not ever use particular mathematics the same way that we do in class, but the mathematical concepts are applicable everywhere and every day. As an added bonus, I also include that they’re exercising their brain. Statistics is a course that I have never taught before, but throughout teaching algebra and geometry, we do address some statistical concepts. Up until this point in my career, teaching statistical concepts has always been a very superficial unit that lacked meaning and application. Every day in our world, people are affected by situations that they feel they have no control over. On a larger scale, injustices could involve distribution of wealth, opportunities, and privileges within a society. In a mathematics classroom, by examining and bringing awareness to social justice issues, several statistical concepts can be explored.

When incorporating social justice into your classroom, there are several key things to keep in mind. First, we all view every social justice issue through the lens of our own experiences. We each have our own backgrounds, biases, and beliefs, and these can sometimes cause us to have tunnel vision and inhibit our abilities to grow and learn. Allowing students to become aware of their own lenses provides a platform for rich classroom conversations in which we can understand issues on a much deeper level. Second, we need to be aware that not all students feel the same way about social justice issues. This isn’t a problem and, in fact, can lead to meaningful discussions; however, students need to be taught how to respectfully discuss issues with others who don’t share their viewpoints. Next, as the teacher, make sure you familiarize yourself with the material before teaching. Your purpose is to serve as a facilitator of conversation, so you need to be aware of the topics, teaching materials, and other issues so that students don’t stumble onto inappropriate content. Lastly, be sure to inform your administrator because social justice topics can be quite controversial, and then he or she will be in the know if a concerned parent calls.

Aside from the social justice awareness that occurs through the use of these types of lessons, students also are involved in furthering their critical-thinking abilities and their statistical literacy. The Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report (Franklin et al., 2007) examines four different process components: (1) “Formulate Question,” (2) “Collect Data,” (3) “Analyze Data,” and (4) “Interpret Results.” Each of these components is then broken up into three levels (a, b, and c) that are based on students’ development in statistical literacy (Figure 1). The social justice lesson provided here focuses on how society impacts our perception of body image. In this lesson, students examine the proportions of several Barbie dolls and action figures to decide whether or not these dolls and action figures are representative of the average man or woman. Students are then provided with a variety of data from which they will formulate their own research question (Formulate Question—Level C). After formulating their research question, students will collect data (Collect Data—Level A/B) from the sources provided to analyze their data from various groups (Analyze Data—Level B/C). Lastly, students will interpret their results and apply what they’ve found to the effects of media influence on social expectations (Interpret Results—Level C). Depending on the age-level of your students, this lesson may need to be adjusted so that students stay within the realm of appropriate data because we are examining body image.

Task

The Barbie vs. the World activity is broken up into 3 days, but more days may be needed, depending on your class. The materials used for this lesson can be found using the following links


On Day 1, students are introduced to the main idea of

The author would first like to thank EDT 663 (Miami University, Summer 2016) for their feedback and support throughout the lesson planning process, especially Stephanie Bradford, Courtney Frydryk, and Erin Magness.
this lesson, which serves the purpose of peaking student interest. On this day, students will work in groups to compare the body measurements of various dolls or action figures to those measurements of the average female or male and various celebrities. Students will then create a graphical representation of their choosing with the intent of analyzing and interpreting their data (Figure 2). Students can then reflect on the data and think critically about the overarching topic of body image.

On Day 2 of the lesson, students will participate in a classroom discussion focusing on their thoughts and findings from Day 1. The teacher will also show articles to the class: The Scary Reality of a Real-Life

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<tr>
<td>I. Formulate question</td>
<td>• Beginning awareness of the statistics question distinction&lt;br&gt;• Teachers pose questions of interest&lt;br&gt;• Questions restricted to the classroom</td>
<td>• Increased awareness of the statistics question distinction&lt;br&gt;• Students begin to pose their own questions of interest&lt;br&gt;• Questions not restricted to the classroom</td>
<td>• Students can make the statistics question distinction&lt;br&gt;• Students pose their own questions of interest&lt;br&gt;• Questions seek generalization</td>
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<td>II. Collect data</td>
<td>• Do not yet design for differences&lt;br&gt;• Census of classroom&lt;br&gt;• Simple experiment</td>
<td>• Beginning awareness of design for differences&lt;br&gt;• Sample surveys; begin to use random selection&lt;br&gt;• Comparative experiment; begin to use random allocation</td>
<td>• Students make design for differences&lt;br&gt;• Sampling designs with random selection&lt;br&gt;• Experimental designs with randomization</td>
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<td>III. Analyze data</td>
<td>• Use particular properties of distributions in the context of a specific example&lt;br&gt;• Display variability within a group&lt;br&gt;• Compare individual to individual&lt;br&gt;• Compare individual to group&lt;br&gt;• Beginning awareness of group to group&lt;br&gt;• Observe association between two variables</td>
<td>• Learn to use particular properties of distributions as tools of analysis&lt;br&gt;• Quantify variability within a group&lt;br&gt;• Compare group to group in displays&lt;br&gt;• Acknowledge sampling error&lt;br&gt;• Some quantification of association; simple models for association</td>
<td>• Understand and use distributions in analysis as a global concept&lt;br&gt;• Measure variability within a group; measure variability between groups&lt;br&gt;• Compare group to group using displays and measures of variability&lt;br&gt;• Describe and quantify sampling error&lt;br&gt;• Quantification of association; fitting of models for association</td>
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<td>IV. Interpret results</td>
<td>• Students do not look beyond the data&lt;br&gt;• No generalization beyond the classroom&lt;br&gt;• Note difference between two individuals with different conditions&lt;br&gt;• Observe association in displays</td>
<td>• Students acknowledge that looking beyond the data is feasible&lt;br&gt;• Acknowledge that a sample may or may not be representative of the larger population&lt;br&gt;• Note the difference between two groups with different conditions&lt;br&gt;• Aware of distinction between observational study and experiment&lt;br&gt;• Note differences in strength of association&lt;br&gt;• Basic interpretation of models for association&lt;br&gt;• Aware of the distinction between association and cause and effect</td>
<td>• Students are able to look beyond the data in some contexts&lt;br&gt;• Generalize from sample to population&lt;br&gt;• Aware of the effect of randomization on the results of experiments&lt;br&gt;• Understand the difference between observational studies and experiments&lt;br&gt;• Interpret measures of strength of association&lt;br&gt;• Interpret models of association&lt;br&gt;• Distinguish between conclusions from association studies and experiments</td>
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Figure 1. GAISE Framework (Franklin et al., 2007, pp. 14–15).

- Do Mattel and Hasbro have a responsibility to create more realistic dolls?
- What impact do you think social media has on how we view ourselves?
- How do you think most people feel about their bodies?

Day 3 focuses on student research and statistical literacy. Students will examine data from sources and develop a statistical question that could be addressed using the articles provided. Students will then collect data that is appropriate for their research question, analyze the data, and then interpret their results to provide an answer to their question and defend their reasoning.

**Differentiation**

This lesson can be differentiated to accommodate a variety of learning styles in a classroom. First, students may be provided a variety of options for their graphical representation used in interpreting their data, which could include box plots, scatter plots, and histograms. Students may need assistance in developing and interpreting their graphical representations. This can be done on an as needed basis. For groups of students struggling to develop a research question, the teacher can provide a list of research questions from which students can choose as well as the relevant data, if necessary. When students create their graphical representation, they have the option of using technological resources or using pencil and paper.

**Conclusion**

From my perspective, to teach statistics for social justice provides students with an opportunity to learn about aspects of society that they may not have knowledge of or familiarity with. Some students might argue that if it’s not something they currently experience in their lives, then why should they care about it. By teaching our students statistics for social justice, we give them the opportunity to be put in a place that makes them feel uncomfortable. Barbie vs. The World: A Social Justice Lesson allows students to examine society’s effect on one’s personal body perception. This lesson puts students in the shoes of those who experience these social injustices and helps them to realize the wrongdoing that is occurring throughout the world. When students have the opportunity to “experience” social injustices, they feel more compelled to make a change. I am hopeful that students will see the need for awareness of social injustices and will be encouraged to take action in their school, community, and even on a larger scale.

**References**

The Wisconsin Association of Mathematics Teacher Educators: Supporting Teacher Education and Professional Development in the Badger State

Michael D. Steele, University of Wisconsin-Milwaukee

In the fall of 1991, a small group of university faculty members gathered in Baltimore to discuss the need for a professional organization devoted to the needs and goals of mathematics teacher educators. This event was the birth of the Association of Mathematics Teacher Educators (AMTE). This year, as the national association celebrates its 25th anniversary, Wisconsin is launching a state affiliate, WI-AMTE. In this article, I describe the overall mission and current work of the national AMTE organization and the goals and unique features of the Wisconsin affiliate.

Who is a teacher educator?

When you think about your answer to this question, you might think about someone like me—a university professor that teaches methods courses to his secondary mathematics teacher candidates, sends them off to their student teaching placements, and wishes them the best in their action-packed first year of teaching. Certainly, the work that I do as a faculty member in the Department of Curriculum and Instruction at the University of Wisconsin-Milwaukee would qualify me as a teacher educator. But the work I do does not reside solely in my department or even my university, and the education that a teacher experiences over their career is not limited to the experiences in their formal teacher education program.

Before my students reach their methods course sequence, they have already had many experiences that shape their beliefs, knowledge, and identity as a future teacher. The mathematics instructors who teach courses to our elementary and secondary mathematics teacher candidates have a stake in teacher education because they are providing them opportunities to develop their content knowledge and shape their conception about what it means to know and do mathematics. Each student’s experiences using mathematics and mathematical thinking in nonmathematics courses implicates other professors across my university as stakeholders in mathematics teacher education. Even during the teacher preparation sequence, there are many important influences on my well-prepared beginning teachers that extend beyond the mathematics methods courses. The university supervisors (often retired teachers) who observe our student teachers play an important role in shaping teacher candidates’ practices and how they provide mathematical learning opportunities to their students. The mentor teacher that works side by side with a teacher candidate every day has a tremendous influence on that candidate’s knowledge, beliefs, and practices. A teacher’s colleagues in the school and the school’s administration also have an impact on what a beginning teacher knows and is able to do. And when the well-started beginning mathematics teacher obtains their first teaching position, the school and district’s other mathematics teachers, coaches, and professional developers all take on roles in furthering that mathematics teacher’s education, along with any university faculty members that they encounter in future coursework.

Mathematics Teacher Education: A Special Flavor of Mathematics Education

Just as the knowledge and skills needed to teach mathematics are different from the knowledge and skills needed to do mathematics, the knowledge and skills needed to educate mathematics teachers are different from the knowledge and skills needed to teach mathematics. In 1991, a group of about 15 mathematics teacher educators gathered together at the National Council of Teachers of Mathematics regional conference to discuss the need for an organization to support the professional learning and research of mathematics teacher educators. Less than a year later, the Association of Mathematics Teacher Educators (AMTE) was formed, with the goal of providing “a national forum . . . to discuss issues of mutual professional concern [and to] share ideas on effective ways of promoting the NCTM Standards, NCSM and MAA recommendations on teaching school mathematics and developing programs to improve the mathematics education of
practicing and future teachers” (Spikell, 1992, p.1).

In the intervening years, the national AMTE organization has grown to over 1,200 members and is a standing member of the Conference Board of Mathematical Sciences alongside other mathematics professional organizations. The Association hosts an annual meeting that presents cutting-edge research in teacher preparation and professional development that is regularly attended by over half of its membership. Ongoing professional development and discourse for its members comes in the form of quarterly webinars, the AMTE Connections Newsletter, and the publication of the journal *Mathematics Teacher Educator* in collaboration with the National Council of Teachers of Mathematics. *Mathematics Teacher Educator* is a journal focused on the scholarly work of teacher education, including initial teacher preparation and professional development. Articles have included discussions of mathematical explorations in content courses for preservice teachers (Conner, 2013), design principles for mathematics methods courses (Steele & Hillen, 2012), and teacher learning from professional development projects (Edgington, Wilson, Sztajn, & Webb, 2016).

AMTE has also had a significant influence on state and national policy. In 2013, AMTE collaborated with the Brookhill Foundation to develop a set of standards for Elementary Mathematics Specialists (Association of Mathematics Teacher Educators [AMTE], 2013). This resource makes a case for the need for Elementary Mathematics Specialists (EMS) and provides guidance regarding the experiences that should constitute a degree program to develop EMS candidates. The document also provides guidance to states looking to establish an EMS certification. AMTE has conducted reviews of mathematics education doctoral programs, produced a variety of position statements on contemporary issues in mathematics education, and recently put forth a draft set of standards for mathematics teacher preparation (AMTE, 2016). The *AMTE Standards for Mathematics Teacher Preparation* represent an important effort to influence policy discussions on the preparation of teachers and to include all stakeholders (not just faculty members at institutions of higher education) in the discussion of how mathematics teachers are prepared.

Through these endeavors, AMTE has positioned itself as a venue for research, professional learning, and policymaking in mathematics education. Their work influences how beginning mathematics teachers are prepared and how we support the ongoing learning of mathematics teachers throughout their professional lives.

**Wisconsin AMTE: An Important New Partner in Strengthening Mathematics Education**

Here in Wisconsin, we have an extremely strong tradition of supporting student learning of mathematics. Our strong state NCTM affiliate, the Wisconsin Mathematics Council, consistently provides its membership with a wide array of experiences to support good mathematics teaching in the state. The Mathematics Institute of Wisconsin (formerly Brookhill Institute of Mathematics) provides content-focused teacher professional development through the Wisconsin Statewide Mathematics Initiative. Countless public and private universities across the state have degree programs and professional development opportunities for mathematics teachers, some funded through federal monies distributed through the Department of Public Instruction. On the whole, Wisconsin mathematics teachers have a wide array of learning opportunities at their disposal, designed and implemented by a large cast of faculty members, consultants, coaches, and mathematics teaching peers.

What we do not have here in the Badger State at present is an infrastructure to support the professional learning of these mathematics teacher educators. The Wisconsin chapter of AMTE intends to bridge that gap by bringing together university mathematicians, faculty members in schools and colleges of education, district coaches and mathematics coordinators, consultants, and teachers that serve as mentors to discuss how we support mathematics teacher learning from the preservice years through induction and to retirement (and sometimes beyond)! We are excited to bring these groups together to discuss issues of teacher preparation program design, supervising beginning teachers, providing feedback connected to the Wisconsin Educator Effectiveness System that is rich and meaningful, and designing professional development opportunities on large and small scales. We envision WI-AMTE as going far beyond university faculty members—coaches, coordinators, and teachers play important roles in mathematics teacher education. It is critical to the continued success of mathematics teaching and learning in Wisconsin for us to collaborate in support of teacher learning as well...
as student learning.

To launch our new organization, we held important events this spring. On January 4, 2017, WI-AMTE hosted a discussion of the AMTE Standards for Mathematics Teacher Preparation and what they mean for Wisconsin. A screencast of this discussion can be viewed on our website: http://wiamte.org/index.php/2017/01/06/webinar-discussion-video/. The AMTE Standards document is an aspirational vision for what teacher preparation could be, and realizing that vision will require significant contributions from all teacher preparation stakeholders.

At the WMC Annual Conference in Green Lake in May, we will host two more events. WI-AMTE will be sponsoring an action research poster session where preservice teachers can share the results of action research investigations they have conducted as a part of their certification programs. Preservice teachers always turn out in significant numbers at Green Lake, and WI-AMTE is happy to pilot this poster session as a way to provide them with a showcase for their own work.

Finally, on Friday, the WMC Annual Conference will feature a WI-AMTE track. In every conference slot that day, WI-AMTE will provide a session for the discussion of issues related to teacher preparation, professional development, and coaching. We will be working with our colleagues at WMC and Brookhill to plan these sessions and support a rich dialogue with a wide array of stakeholders in mathematics teacher education.

In the future, we hope to continue to provide unique events to support mathematics teacher educators and to partner with our colleague organizations in the state to support the needs of mathematics teachers and learners across the state. To learn more about WI-AMTE, visit our fledgling website at wiamte.org, or contact any of our leadership team. That team includes myself, Matthew Chedister and Joshua Hertel from the University of Wisconsin-La Crosse, and Lynn Schaal from the New London School District. WI-AMTE looks forward to working with you to strengthen our extraordinary mathematics education community in Wisconsin!

References


WMC Puzzle Page

Search-A-Word: Dylan Wiliam

U G N I T A V I T C A Y K I T
E N E D E D D D E B M E S R N E
V T D V X K C A B D E E F S C
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State Mathematics Competition

The following problem is from the 2016 High School State Mathematics Contest. For additional questions and solutions, visit http://www.wismath.org/contests.

In simplest form, what is the numerical value of the following expression:
\[
\left(\sqrt{2016}\right)\left(\sqrt{2016}\right)\left(\sqrt{2016}\right)
\]

Ken Ken

Fill in the blank squares so that each row and each column contain all of the digits 1 through 4. The heavy lines indicate areas that contain groups of numbers that can be combined (in any order) to produce the result shown with the indicated math operation.

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Sudoku

Fill in the blank squares so that each row, each column and each 3-by-3 block contain all of the digits 1 through 9.

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You’re in Good Company...

WMC members include all levels of teachers from pre-kindergarten to university instructors, mathematics teacher leaders, administrators, and mathematics specialists. Anyone whose goal is to improve mathematics education in his or her classroom, building, district, or state is welcome to join.

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- Professional Development
- Publications
- Recognition
- Student Activities
- Networking

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