Unfolding Mathematics with Unit Origami

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http://piman1.wikispaces.com
Assumptions:

mathematics is the search for patterns-
patterns come from problems-
therefore, mathematics is problem solving.

algebra is the language of mathematics

knowing and describing change is important

tables, graphs, equations, words are effective ways to describe change

math should be fun
Example-The Ripple Effect

How do the number of connections change as the number of people grows?
Two people.
One connection.

Three people.
Three connections.

Four people.
? connections.
Table
Graph

A = (2, 1)
B = (3, 3)
C = (4, 6)
D = (5, 10)
E = (6, 15)
F = (7, 21)
G = (8, 28)
Verbal explanation:

In a group of 10 people, each of the 10 could be connected to 9 others.

Those connections are all counted twice—me to you and you to me.

Therefore the number of connections for 10 people is
In General:

connections = \frac{\text{people} \times (\text{people} - 1)}{2}
This cube was made from 6 squares of paper that were 8 inches on each side.
Here are some other cubes, using the same unit, but starting with square paper of other sizes.

The volume or size of each cube changes as the size of the square that was folded changes.
here is the paper that was used-

the paper ranges in size from a 3" square to an 8" square
First, we are going to build a cube.

This process is called multidimensional transformation because we transform square paper into a three dimensional cube.

Another more common name is

UNIT ORIGAMI
two very useful books—highly recommended.

Unit Origami,
Tomoko Fuse

Unfolding Mathematics with Unit Origami,
Key Curriculum Press
Start with a square.
Fold it in half, then unfold.
Fold the two vertical edges to the middle to construct these lines which divide the paper into fourths. Then unfold as shown here.
Fold the lower right and upper left corners to the line as shown. Stay behind the vertical line a little. You will see why later.
Now, double fold the two corners. Again, stay behind the line.
Refold the two sides back to the midline. Now you see why you needed to stay behind the line a little. If you didn't, things bunch up along the folds.
Fold the upper right and lower left up and down as shown. Your accuracy in folding is shown by how close the two edges in the middle come together. Close is good—not close could be problematic.
The two corners you just folded, tuck them under the double fold. It should look like this.
Turn the unit over so you don't see the double folds.
Lastly, fold the two vertices of the parallelogram up to form this square. You should see the double folds on top.
This is one UNIT.

We need 5 more UNITS to construct a cube.
The volume of the cube will change when different size squares are folded.

The cubes you just made were made from 6” squares.

What if the square was half as long?

Is there a relation between the size of the square and the resulting volume that we could show using a table, graph, or algebraic expression?
Gather Data

Record in a table

Graph on a grid

Determine the equation using geogebra
Gathering Data

<table>
<thead>
<tr>
<th>original square</th>
<th>number of buckets</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
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<tr>
<td>5</td>
<td></td>
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<tr>
<td>5.5</td>
<td></td>
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<tr>
<td>6</td>
<td></td>
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<tr>
<td>7</td>
<td></td>
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<td>8</td>
<td></td>
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<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>
# Gathering Data

<table>
<thead>
<tr>
<th>original square</th>
<th>number of buckets</th>
<th>group 1</th>
<th>group 2</th>
<th>group 3</th>
<th>group 4</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
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<td>3</td>
<td>1.5</td>
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<td>10</td>
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</tbody>
</table>
What is “under” the unit that we just folded. Unfolding it reveals these lines. The center square is the face of the cube. If the square is 8” by 8”, what is the area of the square in the middle?
<table>
<thead>
<tr>
<th>length of original square</th>
<th>resulting length of cube</th>
<th>resulting area of one face of cube</th>
<th>resulting volume of cube</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
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<td>10</td>
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</tr>
<tr>
<td>length of original square</td>
<td>resulting length of cube</td>
<td>resulting area of one face of cube</td>
<td>resulting volume of cube</td>
</tr>
<tr>
<td>---------------------------</td>
<td>--------------------------</td>
<td>-----------------------------------</td>
<td>--------------------------</td>
</tr>
<tr>
<td>2</td>
<td>0.707</td>
<td>0.5</td>
<td>0.354</td>
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<tr>
<td>4</td>
<td>1.414</td>
<td>2</td>
<td>2.828</td>
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<tr>
<td>6</td>
<td>2.121</td>
<td>4.5</td>
<td>9.546</td>
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<tr>
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<td>2.828</td>
<td>8</td>
<td>22.627</td>
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<tr>
<td>10</td>
<td>3.535</td>
<td>12.5</td>
<td>44.194</td>
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</tbody>
</table>

\[
x \quad \sqrt{\frac{x^2}{8}} \quad \frac{x^2}{8} \quad \left(\sqrt{\frac{x^2}{8}}\right)^3
\]
other uses for this unit:

model volume, surface area, and length

Sierpinski’s Carpet in 3 dimensions

model the Painted Cube problem

construct stellated icosahedron with 30 units, stellated octahedron with 12 units

and .......
here is a stellated icosahedron on-
30 units are required
this is a Bucky ball, 270 units
a science fair project—determining how many structures the unit can make
entertaining grandchildren
Sierpinski’s carpet in 3 dimensions-
a model for volume
a wall of cubes!
Have you ever wanted an equilateral triangle?

or

How about a regular hexagon?

or

A tetrahedron?

or

What about a truncated tetrahedron?
start with any rectangular sheet of paper-
fold to find the midline-
fold the lower right corner up as shown-
fold the upper right corner as shown-
fold over the little triangle-
sources that would be helpful:

handout: this keynote is available in pdf form at

http://piman1.wikispaces.com

Unit Origami, Tomoko Fuse

Unfolding Mathematics using Unit Origami, Key Curriculum Press

geogebra.org

Fold In Origami, Unfold Math,
Origami Song

I’d like to teach all kids to fold
And learn geometry
To see how paper can be used
To do origami.

I’d like to build a cube that before
The formula’s applied
To show that volume all depends
On the lengths of that cube’s side

I’d like to start with paper squares
And fold with symmetry
I use right angles here and there
To make my squares 3-D

Chorus:

That’s geometry, we learned here today
A song of math that echoes on and never goes away
A song of math that echoes on and never goes away.