

**PROBLEM-SOLVING SITUATIONS**

<b>JOINING PROBLEMS</b>		
<b>Join: Result Unknown (JRU)</b>	<b>Join: Change Unknown (JCU)</b>	<b>Join: Start Unknown (JSU)</b>
<p>◆ Grandmother had 5 strawberries. Grandfather gave her 8 more strawberries. How many strawberries does Grandmother have now?</p> <p align="center"><math>5 + 8 = \square</math></p>	<p>♥ Grandmother had 5 strawberries. Grandfather gave her some more. Then Grandmother had 13 strawberries. How many strawberries did Grandfather give Grandmother?</p> <p align="center"><math>5 + \square = 13</math></p>	<p>♠ Grandmother had some strawberries, Grandfather gave her 8 more. Then she had 13 strawberries. How many strawberries did Grandmother have before Grandfather gave her any?</p> <p align="center"><math>\square + 8 = 13</math></p>
<b>SEPARATING PROBLEMS</b>		
<b>Separate: Result Unknown (SRU)</b>	<b>Separate: Change Unknown (SCU)</b>	<b>Separate: Start Unknown (SSU)</b>
<p>◆ Grandfather had 13 strawberries. He gave 5 strawberries to Grandmother. How many strawberries does Grandfather have left?</p> <p align="center"><math>13 - 5 = \square</math></p>	<p>♥ Grandfather had 13 strawberries. He gave some to Grandmother. Now he has 5 strawberries left. How many strawberries did Grandfather give Grandmother?</p> <p align="center"><math>13 - \square = 5</math></p>	<p>♠ Grandfather had some strawberries. He gave 5 to Grandmother. Now he has 8 strawberries left. How many strawberries did Grandfather have before he gave any to Grandmother?</p> <p align="center"><math>\square - 5 = 8</math></p>
<b>PART-PART-WHOLE PROBLEMS</b>		
<b>Part-Part-Whole: Whole Unknown (PPW:WU)</b>	<b>Part-Part-Whole: Part Unknown (PPW:PU)</b>	
<p>◆ Grandmother has 5 big strawberries and 8 small strawberries. How many strawberries does Grandmother have altogether?</p> <p align="center"><math>5 + 8 = \square</math></p>	<p>♥ Grandmother has 13 strawberries. Five are big and the rest are small. How many small strawberries does Grandmother have?</p> <p align="center"><math>13 - 5 = \square</math> or <math>5 + \square = 13</math></p>	
<b>COMPARE PROBLEMS</b>		
<b>Comp. Difference Unknown</b>	<b>Comp. Quantity Unknown</b>	<b>Comp. Referent Unknown</b>
<p>◆ ♥ Grandfather has 8 strawberries. Grandmother has 5 strawberries. How many more berries does Grandfather have than Grandmother?</p> <p align="center"><math>8 - 5 = \square</math> or <math>5 + \square = 8</math></p>	<p>♠ Grandmother has 5 strawberries. Grandfather has 3 more strawberries than Grandmother. How many strawberries does Grandfather have?</p> <p align="center"><math>5 + 3 = \square</math></p>	<p>♠ Grandfather has 8 strawberries. He has 3 more strawberries than Grandmother. How many strawberries does Grandmother have?</p> <p align="center"><math>8 - 3 = \square</math> or <math>\square + 3 = 8</math></p>
<b>MULTIPLICATION &amp; DIVISION PROBLEMS</b>		
<b>Multiplication</b>	<b>Measurement Division</b>	<b>Partitive Division</b>
<p>◆ Grandmother has 4 piles of strawberries. There are 3 strawberries in each pile. How many strawberries does Grandmother have?</p> <p align="center"><math>4 \times 3 = \square</math></p>	<p>◆ Grandmother had 12 strawberries. She gave them to some children. She gave each child 3 strawberries. How many children were given strawberries?</p> <p align="center"><math>12 \div 3 = \square</math></p>	<p>◆ ♥ Grandfather has 12 strawberries. He wants to give them to 3 children. If he gives the same number of strawberries to each child, how many strawberries will each child get?</p> <p align="center"><math>12 \div 3 = \square</math></p>

Problem chart based on Cognitively Guided Instruction Problem Types (Carpenter et al., 1996)



# MATH STORY PROBLEM TYPES

## JOINING PROBLEMS

Join (Result Unknown) $6 + 3 = \underline{\quad}$	Join (Change Unknown) $4 + \underline{\quad} = 7$	Join (Start Unknown) $\underline{\quad} + 4 = 6$
Mr. Smith had 6 cookies. Suzy gave him 3 more cookies. How many cookies does Mr. Smith have now?	Mr. Smith had 4 cookies. Suzy gave him some more. Then, Mr. Smith had 7 cookies. How many cookies did Suzy give Mr. Smith?	Mr. Smith had some cookies. Suzy gave him 4 more cookies. Then, he had 6 cookies. How many cookies did Mr. Smith start with?

## SEPARATING PROBLEMS

Separate (Result Unknown) $7 - 4 = \underline{\quad}$	Separate (Change Unknown) $5 - \underline{\quad} = 1$	Separate (Start Unknown) $\underline{\quad} - 4 = 4$
Mr. Smith had 7 cookies. He gave 4 of them to Suzy. How many cookies did Mr. Smith have left?	Mr. Smith had 5 cookies. He gave some to Suzy. Then, he had 1 cookie left. How many cookies did Mr. Smith give to Suzy?	Mr. Smith had some cookies. He gave 4 to Suzy. Then, he had 4 cookies left. How many cookies did Mr. Smith have to start with?

## PART - PART - WHOLE PROBLEMS

Part - Part - Whole (Whole Unknown) $6 + 3 = \underline{\quad}$	Part - Part - Whole (Part Unknown) $7 - 4 = \underline{\quad}$ or $4 + \underline{\quad} = 7$
Mr. Smith had 6 white cookies and 3 pink cookies. How many cookies did Mr. Smith have altogether?	Mr. Smith had 7 cookies. 4 were pink and the rest were white. How many white cookies did Mr. Smith have?

## COMPARING PROBLEMS

Compare (Difference Unknown) $5 - 3 = \underline{\quad}$ or $3 + \underline{\quad} = 5$	Compare (Quantity Unknown) $3 + 2 = \underline{\quad}$	Compare (Referent Unknown) $8 - 5 = \underline{\quad}$
Mr. Smith had 5 cookies. Suzy had 3 cookies. How many more cookies did Mr. Smith have than Suzy?	Mr. Smith had 3 cookies. Suzy had 2 more cookies than Mr. Smith. How many cookies did Suzy have?	Mr. Smith had 8 cookies. He had 5 more than Suzy. How many cookies did Suzy have?

## MULTIPLYING AND DIVIDING PROBLEMS

Multiplication $3 \times 3 = \underline{\quad}$	Measurement Division $9 \div 3 = \underline{\quad}$	Partitive Division $12 \div 3 = \underline{\quad}$
Mr. Smith had 3 piles of cookies. There were 3 cookies in each pile. How many cookies did Mr. Smith have?	Mr. Smith had 9 cookies. He put 3 cookies in each box. How many boxes did he need?	Mr. Smith had 12 cookies. He wanted to give them to 3 friends. How many cookies did each friend get?

# CGI Problem Types

JOIN	Result Unknown	Change Unknown	Start Unknown
SEPARATE	Result Unknown	Change Unknown	Start Unknown
Part-Part-Whole	Whole Unknown		Part Unknown
			Referent Set Unknown
Compare	Difference Unknown	Compare Quantity Unknown	

	Multiplication	Measurement Division	Partitive Division
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When planning the next level of challenge for each student, teachers use the MMSD K-5 Grade Level Mathematics Standards to guide the selection of problem types and number sizes. Teachers know that every student will take a unique path in becoming proficient with the problem types, solution strategies and number sizes. An overview of the K-2 development as described in the MMSD standards appears in the following table:

Kindergarten	First Grade	Second Grade
<p><b>Problem types:</b> Join, Result Unknown Separate, Result Unknown Multiplication Measurement Division Partitive Division, sharing by 2</p> <p><b>Solution strategies:</b> Modeling strategies Acting it out Using objects Drawing pictures Counting strategies, Moving into counting on 1 and 2</p> <p><b>Number Sizes:</b> Modeling strategies: 0 – 20; focus on 0 – 10</p>	<p><b>Problem types:</b> Join and Separate, Result Unknown Multiplication Measurement Division Partitive Division, sharing by 2, 3 and 4 Join and Separate, Change Unknown Compare, Difference Unknown Part, Part, Whole, Whole Unknown</p> <p><b>Solution strategies:</b> Counting strategies Counting on or back 1, 2, 3 Counting by groups of 2, 5, 10 Modeling strategies Acting it out Using objects Drawing pictures Using empty number lines</p> <p><b>Number sizes:</b> Mental computations: Addition - 0 – 10, moving into 0 – 20 Modeling strategies: 0 – 100</p>	<p><b>Problem type:</b> Join and Separate, Result Unknown Multiplication Measurement Division Partitive Division, sharing by 2, 3, 4, 6 and 8 Join and Separate, Change Unknown Compare, Difference Unknown Part, Part, Whole, Whole Unknown Part, Part, Whole, Part Unknown Join and Separate, Start Unknown</p> <p><b>Solution strategies:</b> Counting strategies Counting on or back 1, 2, 3 Counting by groups of 2, 5, 10 Decomposing numbers Using landmarks (10) Using place value concepts Modeling strategies Acting it out Drawing pictures Using objects Using empty number lines Using arrow language</p> <p><b>Number sizes:</b> Mental computations: Addition – Sums from 0 – 20 Place value concepts – 0 – 100 Subtraction – Differences of 1, 2 or 3 Modeling and counting strategies: Numbers beyond 100</p>

Name/Grade

Green Type CGI Problems

Clip Number	Clip 4.1	Clip 3.16	Clip 3.2	Clip 4.5	Clip _____	Clip _____	Clip _____
Problem Type	JRU	JRU	JRU	JRU	JRU	JRU	JRU
	SRU	SRU	SRU	SRU	SRU	SRU	SRU
	Multiplication	Multiplication	Multiplication	Multiplication	Multiplication	Multiplication	Multiplication
	PPW:WU	PPW:WU	PPW:WU	PPW:WU	PPW:WU	PPW:WU	PPW:WU
Strategy Used	No Strategy	No Strategy	No Strategy	No Strategy	No Strategy	No Strategy	No Strategy
	Direct Modeling	Direct Modeling	Direct Modeling	Direct Modeling	Direct Modeling	Direct Modeling	Direct Modeling
	Counting Strategy	Counting Strategy	Counting Strategy	Counting Strategy	Counting Strategy	Counting Strategy	Counting Strategy
	Derived	Derived	Derived	Derived	Derived	Derived	Derived
Notes							

	JRU	SRU	M	PPW:WU
Strategy Used:	No Strategy	No Strategy	No Strategy	No Strategy
	<p><b>Direct Modeling-</b> Using a manipulative, student will make a pile of 5 and a pile of 3, will join them all together and then count each one.</p> <p><b>Counted On-</b> Says 5 or 3, then will count on their fingers the rest of the way. Ex: 5.....6, 7, 8 or 3... 4, 5, 6, 7, 8</p> <p><b>Derived-</b> Knew that 5 + 3 is 8 because they are fluent. When using 15, 13 the student might say "10 + 10 is 20, and 5 + 3 is 8, so 20 and 8 is 28." Able to solve mentally with a strategy.</p>	<p><b>Direct Modeling-</b> Using a manipulative, student will make a pile of 8 and take away 3, then count each one to find the answer.</p> <p><b>Counted back/Added On-</b> Counts back from 8 three times to get 5, or counts back until they get to 3 to get 5.</p> <p><b>Derived-</b> Able to solve mentally. When using bigger numbers, students are able to break apart (decompose) numbers to make the problem easier for them. Ex: "18-10 is 8, then take 3 more away to get 5."</p>	<p><b>Direct Modeling-</b> Using manipulatives, student will create an area for 3 baskets, and put two cubes in each basket. Will then count each cube.</p> <p><b>Skip Counting-</b> Using manipulatives or fingers, student will count by 2's to solve the problem.</p> <p><b>Derived-</b> Able to solve mentally. When using bigger numbers, student will be able to break apart (decompose) numbers to make the problem easier for them. Ex: "5x5 is 25, plus 5 more is 30."</p>	<p><b>Direct Modeling-</b>Using a manipulative, student will make a pile of 5 and a pile of 4, will join them all together and then count each one.</p> <p><b>Counted On-</b> Says 5 or 4, then will count on their fingers the rest of the way. Ex: 5.....6, 7, 8, 9 or 4...5, 6, 7, 8, 9</p> <p><b>Derived-</b>Knew that 5 + 4 is 8 because they are fluent. When using 15, 14 the student might say "10 + 10 is 20, and 5 + 4 is 9, so 20 and 9 is 29." Able to solve mentally with a strategy.</p>