WISCONSIN MIDDLE SCHOOL STATE MATHEMATICS MEET
WISCONSIN MATHEMATICS COUNCIL
March 1 – 5, 2021

Solutions

Problem set #1

1.
$$\frac{4+x}{5+x}=\frac{8+x}{10+x}$$
$$(4+x)(10+x)=(5+x)(8+x)$$
$$40+14x+x^2=40+13x+x^2$$
$$40+14x=40+13x$$
$$14x=13x$$
$$x=\boxed{0}$$

2. Let n be the number of nickels you have. Write: $\$0.05n+\$0.10(2n)+\$0.25(2(2n))=\$7.50$
$$\$1.25n=\$7.50$$
$$n=6$$
Therefore, there are $\boxed{6}$ nickels in the pile of coins.

3. First of all, it is easy to see that $n = 2$ is the smallest n can be. The trick is finding the largest value for n. Note that $\frac{2019}{2021}\approx 0.9990104$. If we subtract 1 from the numerator and the denominator, we get a slightly smaller fraction: $\frac{2018}{2020}=\frac{1009}{1010}\approx 0.9990099$, which is as close to $\frac{2019}{2021}$ as you can get. Since $2 \le n \le 1009$, there are $\boxed{1008}$ possible values for n. You can also arrive at this solution with guess-and-check, if you do not recognize that adding a small amount to both the numerator and the denominator changes the value of the fraction slightly.

Problem set #2

1. $\frac{1\text{ chuckle}}{3\text{ giggles}}\cdot\frac{1\text{ laugh}}{5\text{ chuckles}}\cdot 315\text{ giggles}=\frac{315}{15}\text{ laughs}=\boxed{\textbf{21 laughs}}$

2. Four pirates digging eight holes is like one pirate digging two holes, but four of them working side by side digging their own holes. Therefore, it is equivalent to saying one pirate digging two holes in 12 hours, which makes six hours per hole. Eight pirates digging 32 holes is like each pirate digging four holes, and each hole takes six hours to dig, so the total time it takes them to dig all these holes is $\frac{6\text{ hours}}{\text{hole}}\cdot 4\text{ holes}=\boxed{\textbf{24 hours}}$.

3. There are two ways to solve this problem. First, recognize that if the dimensions of an object are x by y by z, then if a similar object has dimensions the same as the original, but multiplied by a scale factor k, then the lengths of the similar object will have dimensions kx by ky by kz. As a result, the surface area of the similar object will be the same as the original object, except that it is multiplied by a factor of k^2. Same goes for the volume of the two objects, except that the volume of the similar object is multiplied by a factor of k^3.

   Look at the dimensions of the two aquariums. The ratios of their side lengths are all the same: $k = \frac{60}{24} = \frac{70}{28} = \frac{165}{66} = \frac{5}{2}$. So to get the volume of the larger aquarium, multiply the volume of the original aquarium by the cube of the scale factor k: $192 \text{ gallons} \cdot \left(\frac{5}{2}\right)^3 = \boxed{\textbf{3,000 gallons}}$.

   The second way to solve this problem is to see how many cubic inches make one gallon of water. There are $24 \cdot 28 \cdot 66 = 44{,}352$ cubic inches in the aquarium of 192 gallons, which is $\frac{44{,}352 \text{ in}^3}{192 \text{ gallons}} = 231 \frac{\text{in}^3}{\text{gal}}$. The volume of the larger aquarium is $60 \cdot 70 \cdot 165 = 693{,}000$ cubic inches, so the number of gallons of water it holds is $\frac{693{,}000 \text{ in}^3}{231 \frac{\text{in}^3}{\text{gal}}} = \boxed{\textbf{3,000 gallons}}$.

Problem set #3

1. Let the cost of the case be c dollars. The cost of the iPod then is ($5c$ – 11) dollars. The total cost of both items is $c + (5c - 11) = \$229$

$$6c - 11 = \$229$$
$$6c = \$240$$
$$c = \boxed{\textbf{\$40}}$$

2. If the first and the third sentences are combined, then we will know how much all four people weight together, which is $121 + 83 = 204$ pounds. We know how much John and Suzy weigh together, so if we subtract their weight from the weight of the group, we will have Derek and Jillian's weight together: $204 - 113 = \boxed{\textbf{91 pounds}}$.

3. This problem is set up to write three equations with three unknowns. Let a = # of points for a 1st choice, b = # of points for a 2nd choice, and c = # of points for a 3rd choice. The system of equations is this:

$$\begin{aligned} 18a + 6b + 3c &= 129 \\ 6a + 12b + 9c &= 81 \\ 3a + 9b + 15c &= 60 \end{aligned}$$

$a = \boxed{\textbf{6 points for 1}^{\textbf{st}}\textbf{ choice}}$, $b = \boxed{\textbf{3 points for 2}^{\textbf{nd}}\textbf{ choice}}$, and $c = \boxed{\textbf{1 point for 3}^{\textbf{rd}}\textbf{ choice}}$.

Problem set #4

1. As every geometry student should know, if two parallel lines are intersected by a transversal, then corresponding angles are congruent to each other. This means that the measures of their angles are equal. Write this equation and solve: $3x+41=8x+6$

$$35=5x$$

$$7=x$$

If 7 = x, then $m\angle 2=8(7)+6=62^\circ$. Also, every geometry student should know that if two lines intersect each other, then the vertical angles they form are congruent to each other. This means that $m\angle 3=\boxed{\mathbf{62^o}}$.

2. Step 1: Separate the overlapping triangles from each other. You will see that the (upper) smaller triangle has a side length of 5 on the left and 9 along the bottom. The larger triangle has a side length of $(5+x)$ on the left and 12 on the bottom. If two triangles are similar, as these two are, then corresponding sides are proportional to each other. Step 2: Set up an equation relating the lengths of their sides: $\frac{5}{5+x}=\frac{9}{12}$

$$60=45+9x$$

$$15=9x$$

$$\boxed{\mathbf{\frac{5}{3}}}=x$$

3. When doing reflections and translations, do them one point at a time, rather than the figure as a whole. When you are finished, the resultant figure should be congruent to the original, because reflections and translations do not change the shape of the figure.

Start with point A. After being reflected over the first line, it ends up at (3, 0). Translating up three units brings it to (3, 3). Finally, reflection over the second line puts $A'=\boxed{\mathbf{(-2, -2)}}$.
Next, work with point B. The first reflection puts it at (-1, 1). Translating up three units puts it at (-1, 4). Finally, reflection over the second line puts $B'=\boxed{\mathbf{(-3, 2)}}$.
Finally, work with point C. After being reflected over the first line, it ends up at (2, 2). Translating up three units brings it to (2, 5). Finally, reflection over the second line puts the point $C'=\boxed{\mathbf{(-4, -1)}}$.

An alternate method of solving this problem, albeit a little more mathy, involves recognizing that the two lines of reflection intersect at (1, 0), and then translate the original triangle to the left one unit (shifting the alternate coordinate system to the right one unit to put the origin at the intersection point (1, 0) is like moving everything that already exists to the left one unit). Then the reflections are over the lines $y = x$ and $y = -x$, which is easier to calculate. When finished, you have to undo the initial translation (move right one unit) to get the final coordinates of the new triangle.

Problem set #5

1. This is a straight Pythagorean Theorem application.

$$50^2 + 120^2 = d^2$$
$$2500 + 14400 = d^2$$
$$16900 = d^2$$
$$130 = d$$

As the crow flies, Wisconsin Rapids is **130 miles** away from Sheboygan.

2. Another straight application of the Pythagorean Theorem.

$$8^2 + 12^2 = d^2$$
$$64 + 144 = d^2$$
$$208 = d^2$$
$$14.4222 = d$$

So the diagonal is between 14 and 15 feet long. How to find how many inches in between it is? Take the decimal portion of the answer and multiply it by 12 inches per foot to get the final answer of **14 ft 5 in**.

3. Let x = the distance the northbound car travels (ultimately, what we want to solve for). Then $x - 1$ = the distance the eastbound car travels. Set up a Pythagorean Theorem equation:

$$x^2 + (x-1)^2 = 29^2$$
$$x^2 + x^2 - 2x + 1 = 841$$
$$2x^2 - 2x = 840$$
$$x^2 - x = 420$$
$$x^2 - x - 420 = 0$$
$$(x-21)(x+20) = 0$$
$x = 21$ or $x = -20$

Since distances cannot be negative, throw out the negative answer. Therefore, the northbound car has traveled **21 miles**.