WISCONSIN HIGH SCHOOL STATE MATHEMATICS MEET

WISCONSIN MATHEMATICS COUNCIL

March 5 – 9, 2018

Problem Set #1 Score: (For Scorer's Use Only)

Name: _____ Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

For this first problem set, calculators are <u>not</u> allowed. They may be used for the remainder of the meet only, starting with Problem Set #2.

Answers

1. (1 point)

   A 6 ft. ladder is leaning against a wall, and the ground distance from the bottom of the ladder to the wall is 3 ft. How far up the wall does the ladder reach? Write your answer in simplest radical form. _____

2. (3 points)

   $\sqrt{4+2\sqrt{3}}-\sqrt{4-2\sqrt{3}}=?$ _____

3. (5 points)

   Find all real numbers x between 0 and 2π inclusive, such that $\tan 7x-\sin 6x=\cos 4x-\cot 7x$. _____

WISCONSIN HIGH SCHOOL STATE MATHEMATICS MEET

WISCONSIN MATHEMATICS COUNCIL

March 5 – 9, 2018

Problem Set #2 Score: (For Scorer's Use Only)

Name: _____ Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

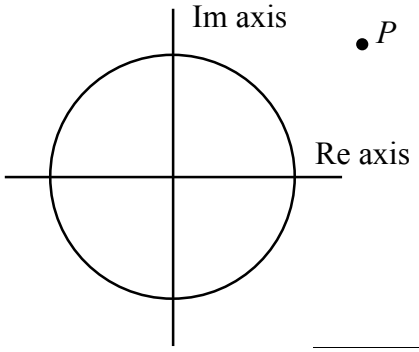
Answers

1. (1 point)

   If $\log_b 2 = x$ and $\log_b 5 = y$, write $\log_b 20^{18}$ in terms of x and y. _____

2. (3 points)

   A unit circle, centered at the origin of the complex plane, is drawn as shown in the diagram. The point P is graphed in the 1st quadrant outside of the circle. Describe where the reciprocal of P should be graphed with respect to the quadrant and the circle.



   _____

3. (5 points)

   What is the ordered pair of real numbers (x, y) for which $16^x - 16^y = 192$ and $4^x - 4^y = 8$? _____

WISCONSIN HIGH SCHOOL STATE MATHEMATICS MEET

WISCONSIN MATHEMATICS COUNCIL

March 5 – 9, 2018

Problem Set #3 Score: (For Scorer's Use Only)

Name: _____ Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

Answers

1. (1 point)

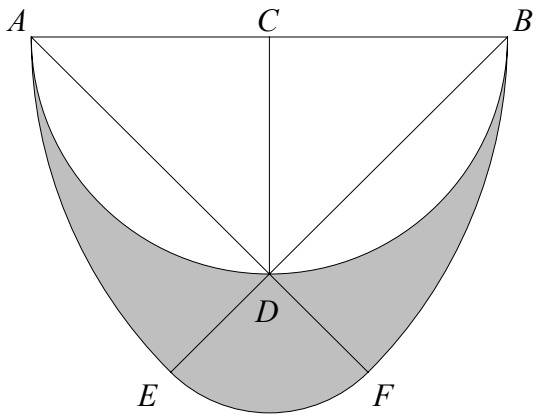
   In a list of seven positive integers (not necessarily distinct), six of them are 7, 10, 15, 18, 21, and 21. Determine the sum of the possible distinct medians of this list. _____

2. (3 points)

   What are the last three digits of 7^{2018}? _____

3. (5 points)

   Semicircle $\overset{\frown}{AB}$ has center C and radius 1. Point D is on $\overset{\frown}{AB}$, and $\overline{CD} \perp \overline{AB}$. Extend $\overline{BD}$ and $\overline{AD}$ to E and F, respectively, so that circular arcs $\overset{\frown}{AE}$ and $\overset{\frown}{BF}$ have B and A as their respective centers. Circular arc $\overset{\frown}{EF}$ has center D. What is the exact area of the shaded "smile" $AEFBDA$?



_____

WISCONSIN HIGH SCHOOL STATE MATHEMATICS MEET

WISCONSIN MATHEMATICS COUNCIL

March 5 – 9, 2018

Problem Set #4 Score: (For Scorer's Use Only)

Name: _____ Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

Answers

1. (1 point)

   Let x be a real number such that $\sec x - \tan x = 2$.
   Evaluate $\sec x + \tan x$. _____

2. (3 points)

   The Dread Pirate Roberts challenges Jack Sparrow to a gambling game with the following rules: Jack will flip a coin, and Roberts will bet half of his money. If the coin lands heads up, Roberts wins his wager from Jack. If the coin lands tails up, Roberts loses his wager to Jack. Roberts wins the same number of flips that he loses. If Roberts starts with \$2018, how much should he expect to have left after 50 flips? Round your answer to the nearest quarter. _____

3. (5 points)

   Determine all values of m such that the equation $x^4 - (3m+2)x^2 + m^2 = 0$ has four real roots in arithmetic progression. _____

WISCONSIN HIGH SCHOOL STATE MATHEMATICS MEET

WISCONSIN MATHEMATICS COUNCIL

March 5 – 9, 2018

Team Problem Set (Page 1) Score: (For Scorer's Use Only)

Team: _____ Captain: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

Answers

1. (10 points)

Define the functions $f_2(x)=(1+x+x^2)^2$

$$f_4(x)=(1+x+x^2+x^3+x^4)^4$$

$$f_6(x)=(1+x+x^2+x^3+x^4+x^5+x^6)^6$$

$$g(x)=f_2(x)f_4(x)f_6(x)$$

Hence, $g(x)$ is a 56^{th} degree polynomial, which can be written as $g(x)=a_{56}x^{56}+a_{55}x^{55}+\cdots+a_2x^2+a_1x+a_0$.

Let $A=a_{56}+a_{54}+a_{52}+\cdots+a_2+a_0$ and $B=a_{55}+a_{53}+a_{51}+\cdots+a_3+a_1$.

That is, A is the sum of the even coefficients, and B is the sum of the odd coefficients. Find A and B. $A =$ _____

$B =$ _____

2. (10 points)

Three students combined their skills to solve 150 problems from a textbook so that each of them separately solved exactly 90 problems. We call a problem "difficult" if it was solved by only one of the students, and we call a problem "easy" if it was solved by all three of them. Let d be the number of difficult problems, and let e be the number of easy problems. Calculate $d - e$. _____

Team Problem Set (Page 2)

3. (10 points)

A circle inscribed in ΔABC touches $\overline{AB}$ at point D so that $AD = 5$ and $DB = 3$. Find BC if $m\angle A = 60^\circ$. _____

4. (10 points)

Let A and B be integers, with $A < B$. Find the fraction $\frac{A}{B}$ with $A+B$ as small as possible such that $\frac{7}{10} < \frac{A}{B} < \frac{11}{15}$. _____

5. (10 points)

The increasing sequence of numbers 1, 5, 6, 25, 26, 30, 31, 125, 126, 130, 131, … is formed with positive integers that are powers of 5 or sums of different powers of 5. Determine the 100^{th} term of this sequence. _____

6. (10 points)

Rectangle $ABCD$ comprises four polygons that can be rearranged to make a square. What is the perimeter of that square? _____