WISCONSIN MIDDLE SCHOOL STATE MATHEMATICS MEET
WISCONSIN MATHEMATICS COUNCIL
March 3 – 7, 2014

Solutions

Problem set #1

1. Let x be the smallest number. Then $x + 1$, $x + 2$, and $x + 3$ are the next three numbers.

$$\begin{aligned} x+(x+1)+(x+2)+(x+3) &= 2014 \\ 4x+6 &= 2014 \\ 4x &= 2008 \\ x &= \boxed{\mathbf{502}} \end{aligned}$$

2. Changing the words into an equation and solving, we have:

$$\begin{aligned} 5(x+3) &= 4(x+4)+8 \\ 5x+15 &= 4x+16+8 \\ 5x+15 &= 4x+24 \\ 5x &= 4x+9 \\ x &= \boxed{\mathbf{9}} \end{aligned}$$

3. If we divide 38 by 135, we get the decimal equivalent of $0.2\overline{814}$. After the 2, we get a three-digit repeating pattern. Another way to look at the repeating pattern is like this: $0.281\overline{481}$. The reason to do this is because the repeating pattern is three digits long, so we can group the decimal part in groups of three digits. If we are looking for the 2014th decimal place, then we count the groups of three digits until we reach the 2014th place and determine if the 2014th digit will be the first, the second, or the third digit in that group. Because 2014 ÷ 3 = 671 R1, the 2014th digit is the first digit in the group (the remainder). With the digits being 4, 8, and 1, the answer we are looking for is the first digit of this group, $\boxed{\mathbf{4.}}$

Problem set #2

1. If the total area of the big rectangle is 336 square inches, then each individual rectangle has an area of 48 square inches. The ratio of the side lengths of each smaller rectangle is 3 to 4 (3 rectangle widths are the same as 4 rectangle lengths). It is easy to see that if the lengths of the sides of the rectangles are 6 inches and 8 inches, then the area is 48 square inches, while still maintaining the 3 to 4 ratio of side lengths. Therefore, to find the total perimeter of the larger rectangle, we have five longer sides and six shorter sides, for a total perimeter of $5(8)+6(6)=\boxed{\textbf{76 inches.}}$

2. We need to construct the sequence of terms from the 1st to the 8th in order to get the product of the numbers asked for. Because each term is the sum of the two previous terms, we can get the 5th one from the 4th and 6th ones: $-4 + x_5 = 4$, or $x_5 = 8$. The subscript number denotes

which term in this sequence we are solving for. Working backwards in this manner towards the first term, we get $x_3 = 12$, $x_2 = -16$, and $x_1 = 28$. Working forwards, we get $x_7 = 12$ and $x_8 = 16$. So the product of the 1st and 8th terms of the sequence is $28 \cdot 16 = \boxed{\mathbf{448.}}$

3. Because the ↑ symbol was defined as raising a number to a power, we can rewrite the equation like this: $4 \uparrow 3^2 \div 4^3 \uparrow 2 = 4 \uparrow 3^x$

$$4^9 \div \left(4^3\right)^2 = 4^{3^x}$$
$$\frac{4^9}{4^6} = 4^{3^x}$$
$$4^3 = 4^{3^x}$$
$$3 = 3^x$$
$$x = \boxed{\mathbf{1}}$$

Problem set #3

1. Set up an equation using the average bowling score and the scores already obtained, then solve it for the missing score: $\dfrac{125+155+165+185+x}{5} = 150$

$$630 + x = 750$$
$$x = \boxed{\mathbf{120}}$$

2. When the big hand points at the 8, it is 30˚ below the 9. Because 40 minutes is two-thirds of one hour, the little hand has moved two-thirds the way from the 9 to the 10. Since between each number on the clock is 30˚, the little hand has moved 20˚ up from the 9, giving a total angle of 30˚ + 20˚ = $\boxed{\mathbf{50^\circ.}}$

3. The side length of the largest cube that will fit inside the box is the greatest common factor of 115, 92, and 161. Factoring these three numbers, we get 115 = 5 · 23, 92 = 2 · 2 · 23, and 161 = 7 · 23. It is easy to see that the GCF of these three numbers is 23. This means that the box can fit five cubes long, four cubes tall, and seven cubes deep. Therefore, the box will hold $5 \cdot 4 \cdot 7 = \boxed{\textbf{140 cubes}}$ in all.

Problem set #4

1. Besides a 2 and a 5 (each number in the description of *T* ends in a zero), the prime factors of the numbers listed, from smallest to largest, are 2, 3, 5, 7, 11, and 13. The next largest prime number (which happens to be missing from the list) is $\boxed{\mathbf{17.}}$

2. Let x = the amount of the paycheck taken before taxes are taken out. The question asks, "\$265.20 is 34% of what?" We set up an equation to solve: $\$265.20 = .34x$

$$\frac{\$265.20}{.34} = x$$

$$\boxed{\mathbf{\$780.00}} = x$$

3. There is a function called <u>combinations</u>, where you take a certain number of objects from a larger group of objects. It looks like this: $C(n,r) = \frac{n!}{r!(n-r)!}$, where there is a group of n objects, and you are selecting r of those n objects. (We saw a problem like this last year.) In this problem, we need to choose one square in each row, with decreasing options as we go. In the first row, we can choose any of the 4 squares. In the row below, we can select 3 of the 4 squares (leaving out the column that we chose in the first step). In the third row, we have our choice of 2 squares, and in the last row, there is 1 square left. Multiplying these numbers together, the total number of desired choices is $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$ choices we can make.

   Now we look at the total number of ways we can choose <u>any</u> 4 squares out of the 16 squares in the grid. It is almost as simple as multiplying $16 \cdot 15 \cdot 14 \cdot 13$. Because it does not matter in what order these squares are chosen, we need to use the combinations function: $C(16, 4)$. Plugging the numbers into this formula, it looks like this: $C(16,4) = \frac{16!}{4!12!}$. Take the number of desired choices divided by the number of possible choices to get the probability we are looking for: $\frac{4!}{C(16,4)} = \frac{4!4!12!}{16!} = \frac{4!4!}{16 \cdot 15 \cdot 14 \cdot 13} = \frac{3!3!}{15 \cdot 14 \cdot 13} = \boxed{\mathbf{\frac{6}{455}}}$

Team Problem set

1. Each of the single digits counts as a palindrome (9), as does each double-digit number that is a multiple of 11 (9 more). For the three-digit numbers, numbers of the form 1?1 are palindromes (10 more), as are 2?2, 3?3, etc. (10 more, 8 times) The total comes to **108.**

2. Let the time to get to the mall at 55 mph be x hours. The time to get there at 35 mph will then be 30 minutes longer, or $x + .5$ hours. We use the equation rate × time = distance to find the time. The distance is the same, so we set the two rate × time parts equal to each other:

$$35(x+.5) = 55x$$
$$35x + 17.5 = 55x$$
$$17.5 = 20x$$
$$\frac{7}{8} = x$$

   Multiply this time times 55 mph to get the distance of $\mathbf{\frac{385}{8}} = \boxed{\mathbf{48\frac{1}{8} = 48.125\ miles.}}$

3. Let s = the length of the side of the square and d = the length of the diagonal. We us the Pythagorean equation to relate the length of the sides to the length of a diagonal: $s^2 + s^2 = d^2$. From this, we write the following equation and solve:

$$\begin{aligned} 4s^2 + 2d^2 &= 400 \\ 4s^2 + 2(2s^2) &= 400 \\ 8s^2 &= 400 \\ s^2 &= 50 \end{aligned}$$

Therefore, the area of the square is $s^2 = \boxed{\mathbf{50\ cm^2}.}$

4. In a geometric sequence, the ratio of one term to the previous term is the same for every term in the sequence. Thus we write the following equation and solve it to get what that ratio is:

$$\begin{aligned} \frac{2x+2}{x} &= \frac{3x+3}{2x+2} \\ (2x+2)(2x+2) &= x(3x+3) \\ 4x^2 + 8x + 4 &= 3x^2 + 3x \\ x^2 + 5x + 4 &= 0 \\ (x+1)(x+4) &= 0 \end{aligned}$$

This means that either x = -1 or x = -4. If x = -1, then plugging this into the above equation shows that the ratio is 0, which does not work. If x = -4, then the common ratio turns out to be $\frac{3}{2}$. The 3rd term of this sequence is -9, which makes the 4th term of the sequence $\boxed{-\frac{\mathbf{27}}{\mathbf{2}}.}$

5. There are two cases to consider: Angie retains the red card the whole time, or the red card gets passed around the table, and Angie ends up with the card. Let's look at case 1 first.

If Angie retains the card the whole time, then there is a $\frac{1}{2}$ chance that she retains the card after Brenda selects one of her cards, and there is a $\frac{1}{2}$ chance that she retains the card after Eleanor selects one of her cards. Therefore, case 1 gives a $\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$ probability that she keeps the red card.

Case 2 is where that red card gets passed around the table, and back into Angie's hand. There is a $\frac{1}{2}$ chance of this happening each draw, for a probability of $\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{32}$.

Therefore, the total probability is found by adding these two together: $\frac{1}{4} + \frac{1}{32} = \boxed{\frac{\mathbf{9}}{\mathbf{32}}}.$

6. Let C = # of hours for Chris to clean the gymnasium alone and S = # of hours for Scott to clean the gymnasium alone. Therefore, $C = x + 4$ and $S = x + 1$. In one hour, Chris can clean $\frac{1}{x+4}$ of the gymnasium by himself, while Scott can clean $\frac{1}{x+1}$ of the gymnasium by himself. We write the equation $\frac{1}{x+4}+\frac{1}{x+1}=\frac{1}{x}$ to represent how much of the job gets done in 1 hour. The right side of the equation represents how much of the job is completed $\left(x \text{ hours}\cdot\frac{1}{x}\text{ of the job per hour}\right)$ in one hour, while the left side is obtained by adding the contributions of each cleaner, how much they can clean in one hour. Solving this equation gives us the number of hours it takes them to complete the job working together:

$$\frac{1}{x+4}+\frac{1}{x+1}=\frac{1}{x}$$

$$\frac{(x+1)+(x+4)}{(x+4)(x+1)}=\frac{1}{x}$$

$$x\left[(x+1)+(x+4)\right]=(x+4)(x+1)$$

$$2x^2+5x=x^2+5x+4$$

$$x^2-4=0$$

$$x=2 \text{ or } x=-2$$

We cannot have a negative x (no such thing as negative time), so we throw that answer out. $x = 2$, so that means it takes the two boys **2 hours** to clean the gym when working together.

2014 Wisconsin Mathematics Council State Middle School Math Meet Feedback Form

Dear teachers and students:

We at the Wisconsin Mathematics Council would like to get your feedback on this year's State Math Meet. Please take a moment to fill out this form and return it with the other materials that you need to return to WMC. Your input will help to shape the future of these math meets to make them a more enjoyable and enriching experience.

1. Were there any questions you felt were too easy or too difficult? Which one(s) and why?

 ____

 ____

2. Were there questions on math subjects you were expecting to see, but did not? If so, what did you expect to see? ____

3. Were there questions on math subjects you saw but did not expect to see? If so, which ones?

 ____

4. Was there enough time to finish the team questions? ______________________________

5. Did you like the first event, where calculators were not allowed? Were they too easy, too hard, or just the right amount of difficulty? ____________________________________

6. We try to include a wide variety of problems with some interesting ways to solve them. Were there any questions that sparked further discussion among students and teachers as a result of seeing them here? Which one(s)? ____________________________________

 ____

7. Any additional comments you would like to make about this year's problem set or the meet in general: ____

 ____

 ____

Thank you for taking the time to complete this feedback form. Your comments are invaluable to us. We hope to have you compete again with us next year!