WISCONSIN HIGH SCHOOL STATE MATHEMATICS MEET
WISCONSIN MATHEMATICS COUNCIL
March 3 – 7, 2014

Problem Set #1 Score: (For Scorer's Use Only)

Name: _____

Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

For this first problem set, calculators are <u>not</u> allowed. They may be used for the remainder of the meet only, starting with Problem Set #2.

Answers

1. (1 point)

   If $\sqrt[5]{x} = 3$, what is the value of $\sqrt{x}$ in simplest radical form? _____

2. (3 points)

   Compute the value of this expression:

   $$\frac{(2014)^3 - (2000)^3 - (14)^3}{(2014)(2000)(14)}$$ _____

3. (5 points)

   What is the smallest positive angle in <u>degrees</u> that solves the following equation:

   $$\sin 8x \cos 2x = \cos 8x \sin 2x$$ _____

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Problem Set #2 Score: (For Scorer's Use Only)

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[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

Answers

1. (1 point)

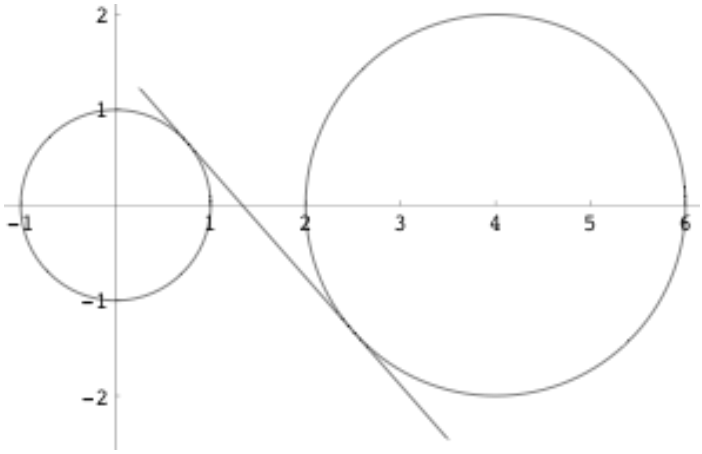
   For all positive numbers a and b, a function f satisfies the equation $f(ab) = f(a) + f(b)$. If $f(5) = x$ and $f(7) = y$, what is the value of $f(245)$? _____

2. (3 points)

   Given that $i^2 = -1$, for how many integers n is $(n + i)^4$ an integer? _____

3. (5 points)

   Find the standard form of the equation of the line that is tangent to both circles as in the displayed picture. _____



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Problem Set #3 Score: (For Scorer's Use Only)

Name: _____

Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

Answers

1. (1 point)

   In ΔABC, $\angle A = 100^\circ$, $\angle B = 50^\circ$, and $\angle C = 30^\circ$. $\overline{AH}$ is an altitude, and $\overline{BM}$ is a median. Find $m\angle MHC$. _____

2. (3 points)

   Scott rolls a red die. His score is equal to the result of the roll. Then Wendy rolls a white die. Her score is the larger of the red die and the white die results. Finally, Deb rolls a blue die. Her score is the largest of the red, the white, and the blue die results. All the dice are fair, 6-sided dice.

   Obviously, Deb's score ≥ Wendy's score ≥ Scott's score. Find the probability that Deb's score > Wendy's score > Scott's score. _____

3. (5 points)

   Find the coefficient of the x^{160} term when the expression $(x^4+1)^{25}(x^5+1)^{20}$ is expanded. _____

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Problem Set #4 Score: (For Scorer's Use Only)

Name: _____

Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

Answers

1. (1 point)

   If $x > 1$ and $y > 1$ and $w = \dfrac{\log_x(\log_x y)}{\log_x y}$, then write an expression for y^w in terms of x and y. $y^w =$ _____

2. (3 points)

   There are six colored vases in a row on a shelf, one of which is blue. The green one is not on the far right. The red one is between two others. The white one is immediately to the left of the orange one. The yellow one lies between the red one and the white one but is adjacent to neither of them.

   From left to right, what order are the vases in? Use the first initials of the colors to write your answer, for example, BGORWY. _____

3. (5 points)

   Find the exact sum of the following expression, written as a fraction in lowest terms:

   .1 + .011 + .00111 + .0001111 + … _____

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Team Problem Set (Page 1) Score: (For Scorer's Use Only)

Team: _____

Captain: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

Answers

1. (10 points)

   Let $f(n)$ be the number of ones that occur in the decimal representation of all the numbers from 1 to n. For example, this gives $f(8)=1$, $f(9)=1$, $f(10)=2$, $f(11)=4$, and $f(12)=5$. Determine the value of $f(10^{100})$. _____

2. (10 points)

   Let S be a set of at least two consecutive integers whose sum is 99. How many distinct sets S exist? _____

3. (10 points)

   A lottery gives prizes that are dollar amounts in powers of 11. The possible prizes are \$1, \$11, \$121, \$1,331, \$14,641, and \$161,051. They will give no more than 10 prizes of any particular amount. They give out exactly \$1,111,111. How many prizes do they give out? _____ prizes

Team Problem Set (Page 2)

4. (10 points)

Suppose that the roots of the quadratic equation $x^2 + ax + b = 0$ are $\sin 15^\circ$ and $\cos 15^\circ$. What is the value of $a^4 - b^2$? _____

5. (10 points)

Curve C_1: $x^2 + y^2 - 16x - 20y + 163 = 0$
Curve C_2: $x^2 + y^2 - 8x - 6y + 21 = 0$

Find the standard form of the equation of the curve C_3 whose points (x, y) satisfy: (x, y) is twice as far from curve C_2 as it is from curve C_1. _____

6. (10 points)

Suppose ΔABC has area $\frac{\sqrt{3}-1}{2}$, $AB = \sqrt{3} - 1$, $AC = 2$, and $\angle CAB$ is acute. What is $m\angle ACB$ (in degrees)? _____$^\circ$