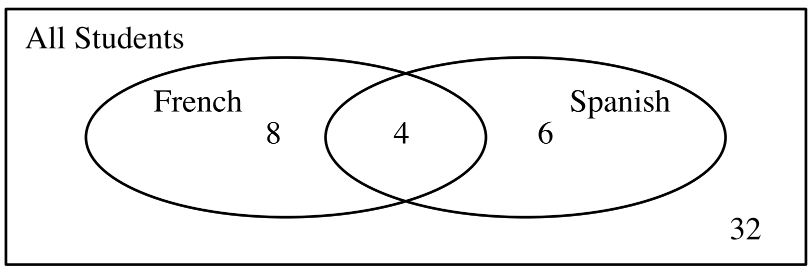
WISCONSIN MIDDLE SCHOOL STATE MATHEMATICS MEET
WISCONSIN MATHEMATICS COUNCIL
March 4 – 8, 2019

Solutions

Problem set #1

1. Both 4^2 and $2^4 = 16$, so $16 + 16 + 16 + 16 = 64$, which is the same as **B) 8^2.**

2. If 10 students take Spanish and 12 students take French, that makes 22 students taking a foreign language. Of those 22 students, 4 take both, so those 4 students have been double-counted. Subtract them from the 22 to get 18 students taking foreign languages. This means that if 18 students are taking foreign languages, **32 students** are taking neither language.



3. You are playing 3 games against each of 6 opponents, for a total of 18 games. Each of your 6 friends do the same thing, for 18 games apiece, or a total of $7 \cdot 18 = 126$ games. However, this double-counts each game played, so divide this by 2 to get **63 games.**

Problem set #2

1. When a single die is rolled, there is a 1 out of 6 chance that it comes up as a 1. For all the dice to show the same number when rolled, multiply this fraction as many times as there are dice: $\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \left(\frac{1}{6}\right)^5 = \frac{1}{7776}$. The answer is **B) $\frac{1}{7776}$.**

2. Let P = how many piglets Farmer Fred bought. This problem can be solved using two equations and two unknowns, but using that method will reduce down to using a single unknown, as seen here:

$$\$40P + \$75(50 - P) = \$2735$$
$$\$3750 - \$35P = \$2735$$
$$\$1015 = \$35P$$
$$\boxed{\mathbf{29}} = P$$

3. If we add 3 students to Mrs. Meyer's class, then no matter if they were separated into groups of 4 or groups of 5 students, there would be none left over. The only way to make sure that

the students can be separated into groups of 4 and groups of 5 is if the number of students was a multiple of the least common multiple of 4 and 5, which is 20. Look at the multiples of 20: 20, 40, 60, 80, …, and subtract the 3 students we added to the class from all of them: 17, 37, 57, 77, …. The smallest number of students from this list is **17 students.**

Problem set #3

1. If the area of each square is 49 square inches, then the side length of the squares is 7 inches. This makes the dimensions of the rectangle 21 inches by 7 inches. The perimeter is the distance around the rectangle, which is $21+7+21+7=56$ inches. The correct answer is **D) 56 inches.**

2. We can get the sum of the test scores from the averages. Cathy's average of 88 on six tests makes the sum of her scores $6 \cdot 88 = 528$. By dropping the lowest test score, the new average is 92 out of 5 tests, for a sum of $5 \cdot 92 = 460$. Subtract these sums to get the score of the test that was dropped: $528 - 460 = \boxed{\mathbf{68.}}$

3. Write an equation to solve this. Let Y = the gift the youngest sibling receives, and each older sibling will get \$5 more than the next youngest one:

$$\begin{aligned} Y+(Y+\$5)+(Y+\$10)+(Y+\$15)+(Y+\$20)+(Y+\$25) &= \$315 \\ 6Y+\$75 &= \$315 \\ 6Y &= \$240 \\ Y &= \boxed{\mathbf{\$40}} \end{aligned}$$

Problem set #4

1. If the product of two numbers is negative, then one of the numbers is positive and the other is negative. Here, we know that x is positive, so y must be negative. Every positive number is greater than every negative number, so the correct choice is $\boxed{\mathbf{D)}\ x > y.}$

2. The average speed for the trip is the total distance traveled divided by the total time spent traveling. The total distance is 240 miles, and Jane spent two hours going to the Dells and three hours on the return trip, for a total of 5 hours. $\dfrac{240 \text{ miles}}{5 \text{ hours}} = \boxed{\mathbf{48\ mph}}$

3. This can be solved by looking at either ΔWLC or ΔGMC. This solution will involve ΔWLC, but by symmetry, it can be solved the other way just the same.

   $m\angle MIL = 124^\circ$, so $m\angle WIM = 56^\circ$. The angles of ΔWIM add to 180°, so $m\angle WMI = 87^\circ$. This makes $m\angle IMC = 93^\circ$. The angles of ΔGMC also add to 180°, so $m\angle WCG = \boxed{\mathbf{45^\circ.}}$

Team Problem set

1. 10 ♣ 10 = $\dfrac{10\cdot 10}{10+10}=\dfrac{100}{20}=5$. 6 ♣ 3 = $\dfrac{6\cdot 3}{6+3}=\dfrac{18}{9}=2$. 5 ♣ 2 = $\dfrac{5\cdot 2}{5+2}=\boxed{\mathbf{\dfrac{10}{7}.}}$

2. In one day, Jack can paint $\frac{1}{20}$ of the ship. In the same amount of time, you can paint $\frac{1}{12}$ of the ship. Together, the two of you can paint $\dfrac{1}{20}+\dfrac{1}{12}=\dfrac{3}{60}+\dfrac{5}{60}=\dfrac{8}{60}=\dfrac{2}{15}$ of the ship in one day. To finish the job, it will take $\dfrac{2}{15}t=1 \rightarrow t=\boxed{\mathbf{\dfrac{15}{2}=7.5\text{ days}}}$ to paint the ship.

3. The length *AB* is 30 units, so one rectangle is 15 units long. There are 5 rectangle widths equal to *CD*, so the rectangles are 6 units wide. The perimeter of ABCD contains 4 lengths and 9 widths, or $4(15)+9(6)=\boxed{\textbf{114 units.}}$

4. The first triangle takes 3 toothpicks. Each extra triangle takes 2 extra toothpicks. A general formula for the number of toothpicks needed to make *N* triangles can be written as $3+2(N-1)$, or more cleanly, $1 + 2N$. For 50 triangles, you need $1+2(50)=\boxed{\textbf{101 toothpicks.}}$

5. TWO + ELEVEN = \$58. Remove letters O, N, and E, worth \$23, and rearrange the remaining letters to make TWELVE: TWØ ~~E~~LEVE~~N~~ → TWELVE = \$58 − \$23 = $\boxed{\textbf{\$35.}}$

6. There is a formula that can be used to find any term in an arithmetic sequence of numbers: $a_n = a_1 + (n-1)d$, where a_n is the nth term in the list, a_1 is the first term, n is the term number being looked for, and d is the common difference between the terms. Here, $a_1 = 9$, $a_n = 533$, and d is 4 (the numbers in the list skip by 4). Solve for n:

$$a_n = a_1 + (n-1)d$$
$$533 = 9 + (n-1)\cdot 4$$
$$533 = 4n + 5$$
$$528 = 4n$$
$$132 = n$$

   Therefore, 533 is the $\boxed{\mathbf{132^{nd}}}$ term of the list.

   If the formula is not known, you can still count by 4s, starting with 9, until you reach 533, and note that you need to count 132 numbers until you reach it.

   Extra credit: If an arithmetic sequence of numbers is formed by adding the same number over and over again to generate a list of numbers, how is a geometric sequence of numbers formed? What would the geometric sequence formula look like?