Scaffolding Thinking: Putting Students' Visual Representations to Work in the Primary Mathematics Classroom

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Join us as we explore...

- connections between MP4: Model with mathematics and MTP3: Use and connect mathematical representations.
- representational competence for teachers and students.
- the role representations play in supporting classroom discussions.
- strategies for engaging young learners in mathematical discussions that scaffold their representational competence.
Learning Targets

• Explore ways to help young learners translate between mathematical representations so they can share their thinking.

• Use young learners’ visual representations as sites for discussions of mathematical ideas.

• Engage in Math Teaching Practice 3: Use and connect mathematical representations.
Representations: Bridges to Modeling with Mathematics
Let’s start with modeling!
MP4: Model with mathematics.

• Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
The Basic Modeling Cycle
p. 59 High School CCSSM

Hmmm...
This is an interesting problem, indeed!

How might I solve it?

I have an answer!

Am I sure?

What does that answer mean?

I need to tell my thinking to someone else.
“As teachers we model with mathematics routinely in our classrooms, but our goal is that our students are also able to model mathematical ideas” (O’Connell & SanGiovanni, 2013, p. 61).

“While teacher modeling [of mathematical ideas] is a powerful instructional tool, our students will only develop this practice if they are creating their own [mathematical] models” (O’Connell & SanGiovanni, 2013, p. 61).
“As teachers we model with mathematics routinely in our classrooms, but our goal is that our students are also able to model mathematical ideas” (O’Connell & SanGiovanni, 2013, p. 61).

“While teacher modeling [of mathematical ideas] is a powerful instructional tool, our students will only develop this practice if they are creating their own (mathematical) models” (O’Connell & SanGiovanni, 2013, p. 61).
The Very Hungry Caterpillar
How much fruit did he eat?

On Monday he ate through 1 apple.
On Tuesday he ate through two pears!
On Thursday he ate through 4 strawberries.
On Friday he ate through 5 oranges.

On Wednesday he ate Through three plums!
Because of the abstract nature of mathematics, people have access to mathematical ideas only through the representations of those ideas.

(National Research Council, 2001, p. 94)
High-leverage Teaching Practice #3

Use and Connect Mathematical Representations

**Physical**: Use concrete objects to show, study, act upon, or manipulate mathematical ideas (e.g., cubes, counters, paper strips).

**Visual**: Illustrate, show, or work with mathematical ideas using diagrams, pictures, number lines, graphs, and other math drawings.

**Symbolic**: Record or work with mathematical ideas using numerals, variables, tables, and other symbols.

**Verbal**: Use language (words) to interpret, state, define, or describe mathematical ideas.

**Contextual**: Situate mathematical ideas in everyday, real-world, imaginary, or mathematical situations and contexts.

Use and Connect
Mathematical Representations

Reference DeAnn’s handout or PtA p. 29

• Person #1 studies the teacher actions.
• Person #2 studies the student actions.

Highlight or mark key ideas on the hand-out. Make note of important actions.

Turn and summarize some of the key ideas with your partner from your respective list.
## Use and Connect Mathematical Representations

### Teacher and Student Actions

<table>
<thead>
<tr>
<th>What are teachers doing?</th>
<th>What are students doing?</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Selecting tasks that allow students to decide which representations to use in making sense of the problems.</td>
<td>• Using multiple forms of representations to make sense of and understand mathematics.</td>
</tr>
<tr>
<td>• Allocating substantial instructional time for students to use, discuss, and make connections among representations.</td>
<td>• Describing and justifying their mathematical understanding and reasoning with drawings, diagrams, and other representations.</td>
</tr>
<tr>
<td>• Introducing forms of representations that can be useful to students.</td>
<td>• Making choices about which forms of representations to use as tools for solving problems.</td>
</tr>
<tr>
<td>• Asking students to make math drawings or use other visual supports to explain and justify their reasoning.</td>
<td>• Sketching diagrams to make sense of problem situations.</td>
</tr>
<tr>
<td>• Focusing students’ attention on the structure structure or essential features of mathematical ideas that appear, regardless of the representation.</td>
<td>• Contextualizing mathematical ideas by connecting them to real-world situations.</td>
</tr>
<tr>
<td>• Designing ways to elicit and assess students’ abilities to use representations meaningfully to solve problems.</td>
<td>• Considering the advantages or suitability of using various representations when solving problems.</td>
</tr>
</tbody>
</table>
...using these different representations is like examining the concept through a variety of lenses, with each lens providing a different perspective that makes the picture (concept) richer and deeper...

Principles to Actions (NCTM, 2014, p. 25)
Students’ Representational Competence

Young learners will:
• Know how and when to use particular mathematical representations
• Self-select representations to use during problem solving.
• Make and explain connections between the representations.

“This implies students view representations as tools they can use to help them solve problems, rather than an end in themselves” (NCTM, 2014, p. 26).
Teachers will:

• Encourage purposeful selection of representations.

• Engage in dialogue about explicit connections among representations.

• Alternate the direction of the connections made among representations.

(NCTM, 2014, p. 26)
Connecting representations to develop representational competence in young learners

Orchestrating discourse after children have worked on problems is particularly important because it is this type of discussion that helps children connect the problem to more general or formal mathematics and make connections to other ideas.

What do representations do?

“In essence, when we ask our students to create mathematical [representations], we challenge them to represent their math understanding—to get it out of their heads” (O’Connell & SanGiovanni, 2013, p. 62).

Representations help students:
• see the problem more clearly.
• visualize the problem.
• simplify the problem.
• make sense of the problem.
• engage in mathematical discourse.
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• engage in mathematical discourse.
On Monday he ate through 1 apple.

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On Friday he ate Through 5 oranges.
“Represent your thinking!”

“I need to see what’s in your head.”

Review the packet of student:

• Identify the representations you see on each piece of student work.
Student A

Student B
Student C

1 2 3 4 5
3 6 10 15

Student D

Jaydon

I
Kowtid 15
Froot
Student G
“But I don’t know what’s in my head.”

How might we put these visual representations “to work” to help a child who does not know what is in his/her head?

Mathematical Goal:
We can show how we count to find a total.

I know the answer is 15. but I don’t know how to show my thinking.
Teachers’ Representational Competence

Teachers will:

- Encourage purposeful selection of representations.
- Engage in dialogue about explicit connections among representations.
- Alternate the direction of the connections made among representations.

(NCTM, 2014, p. 26)
Strategy #1
Encourage purposeful selection of representations.

Read Strategy #1 on Hand-out 2
Turn and share the authors’ message.

Revisit each piece of student work.
• Select 2-3 pieces of student work that honor the context of the story.
• Explain how you would use this work to discuss features of a visual representation that connects with the context of the story.
Teachers’ Representational Competence

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• Encourage purposeful selection of representations.

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(NCTM, 2014, p. 26)
Strategy #2
Engage in dialogue about explicit connections among representations.

Read Strategy #2 Hand-out 2.
Turn and share the authors’ message.

Read the section on
• Select 2-3 pieces of student work that show a range of representations.
• Craft a dialogue discussing the similarities and differences between the representations in the student work.
Teachers will:

• Encourage purposeful selection of representations.

• Engage in dialogue about explicit connections among representations.

• Alternate the direction of the connections made among representations.

(NCTM, 2014, p. 26)
**Strategy #3**
Alternate the direction of connections made among representations

Read Strategy #3 on Hand-out 2. Turn and share the authors’ message.

Create questions that push students to move flexibly between representations.

- Student Work E
- Student Work C (Damain)
- Student Work A (Evan)
- Student Work F (Mikaela)
Alternating Directionality

Step #1
Identify the representations that the student has used. (strength)

Step #2
Use the star model and pick a representation that would deepen a discussion around the work.

Step #3
Craft a question that would facilitate this shift.
Different representations should:

• Be introduced, discussed, and connected;

• Focus students’ attention on the structure or essential features of mathematical ideas; and

• Support students’ ability to justify and explain their reasoning.

*Strengthening the ability to move between and among these representations improves the growth of children’s understanding of mathematical concepts.*

Lesh, Post, & Behr, 1987
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Thanks for coming!
Beth & Melissa

We always enjoy thinking and learning with you!

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