**Physical**: Use concrete objects to show, study, act upon, or manipulate mathematical ideas (e.g., cubes, counters, paper strips).

**Visual**: Illustrate, show, or work with mathematical ideas using diagrams, pictures, number lines, graphs, and other math drawings.

**Symbolic**: Record or work with mathematical ideas using numerals, variables, tables, and other symbols.

**Verbal**: Use language (words) to interpret, state, define, or describe mathematical ideas.

**Contextual**: Situate mathematical ideas in everyday, real-world, imaginary, or mathematical situations and contexts.

---


---

**Use and Connect Mathematical Representations**

*Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.*

Effective mathematics teaching includes a strong focus on using varied mathematical representations. NCTM (2000) highlighted the important role of mathematical representations in the teaching and learning of mathematics by including the Process Standard for Representation in *Principles and Standards for School Mathematics*. Representations embody critical features of mathematical constructs and actions, such as drawing diagrams and using words to show and explain the meaning of fractions, ratios, or the operation of multiplication. When students learn to represent, discuss, and make connections among mathematical ideas in multiple forms, they demonstrate deeper mathematical understanding and enhanced problem-solving abilities (Fuson, Kalchman, and Bransford 2005; Lesh, Post, and Behr 1987).

---

Principles to Actions (NCTM, 2014, p. 24)
Star Students Make Connections
Anne Marie Marshall, Alison Castro Superfine, and Reality Canty

Instructional Strategies To Develop Representational Competence
Teachers can prompt students’ discussions and interactions with representations to enhance their knowledge of the relationships among multiple representations and promote representational competence.

Strategy #1: Encourage purposeful selection of representations
Discuss a variety of reasons to use particular representations, including but not limited to the following: efficiency, accuracy, ease of use, appropriateness with respect to the problem context, and student preference. Different representations often highlight different mathematical concepts or features of a concept. Therefore, it is important for students to understand the purpose for using particular representations. The use of multiple representations during instruction may help students form an understanding of the relative costs and benefits associated with a representation for a given range of situations. Encouraging purposeful selection may help students acquire the competence to consider the ways in which information is conveyed in the representation in relation to what they are being asked to do in the problem. By comparing the use of multiple representations for the same problem, students can more easily see the suitability of one representation over another.

Strategy #2: Engage in dialog about explicit connections among representations.
Allow students to select from a range of representations when solving problems. Also encourage students to try solving the same problem using multiple representations. Then discuss with them the similarities and differences of the multiple representations within the context of one problem.

Strategy #3: Alternate the direction of the connections made among representations.
Help students to move flexibly between representations. Alternating the directionality of the connections made between representations can support this. During instruction, students focus on specific parts of one representation and think about the correspondence with parts in another. The teacher asks questions that require students to translate between representations. Alternating directionality supports students’ thinking with various representations.

References

I love to eat burgers. I love to play soccer. I also love to play piano. I have 15 parrots. I have a dog named Jack. If you have a pet, what is it?