STANDARD 1 Knowledge of Mathematics and General Pedagogy

Teachers of mathematics should have a deep knowledge of—

- sound and significant mathematics;
- theories of student intellectual development across the spectrum of diverse learners;
- modes of instruction and assessment; and
- effective communication and motivational strategies.

STANDARD 2 Knowledge of Student Mathematical Learning

Teachers of mathematics must know and recognize the importance of—

- what is known about the ways students learn mathematics;
- methods of supporting students as they struggle to make sense of mathematical concepts and procedures;
- ways to help students build on informal mathematical understandings;
- a variety of tools for use in mathematical investigation and the benefits and limitations of those tools; and
- ways to stimulate engagement and guide the exploration of the mathematical processes of problem solving, reasoning and proof, communication, connections, and representations.

STANDARD 3 Worthwhile Mathematical Tasks

The teacher of mathematics should design learning experiences and pose tasks that are based on sound and significant mathematics and that—

- engage students' intellect;
- develop mathematical understandings and skills;
- stimulate students to make connections and develop a coherent framework for mathematical ideas;
- call for problem formulation, problem solving, and mathematical reasoning;
- promote communication about mathematics;
- represent mathematics as an ongoing human activity; and
- display sensitivity to, and draw on, students' diverse background experiences and dispositions.

STANDARD 4 Learning Environment

The teacher of mathematics should create a learning environment that provides—

- the time necessary to explore sound mathematics and deal with significant ideas and problems;
- a physical space and appropriate materials that facilitate students' learning of mathematics;
- access and encouragement to use appropriate technology;
- a context that encourages the development of mathematical skill and proficiency;
- an atmosphere of respect and value for students' ideas and ways of thinking;
- an opportunity to work independently or collaboratively to make sense of mathematics;
- a climate for students to take intellectual risks in raising questions and formulating conjectures; and
- encouragement for the student to display a sense of mathematical competence by validating and supporting ideas with a mathematical argument.

STANDARD 5 Discourse

The teacher of mathematics should orchestrate discourse by—

- posing questions and tasks that elicit, engage, and challenge each student's thinking;
- listening carefully to students' ideas and deciding what to pursue in depth from among the ideas that students generate during a discussion;
- asking students to clarify and justify their ideas orally and in writing and by accepting a variety of presentation modes;
- deciding when and how to attach mathematical notation and language to students' ideas;
- encouraging and accepting the use of multiple representations;
- making available tools for exploration and analysis;
- deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let students wrestle with a difficulty; and
- monitoring students' participation in discussions and deciding when and how to encourage each student to participate.

STANDARD 6 Reflection on Student Learning

The teacher of mathematics should engage in ongoing analysis of students' learning by—

- observing, listening to, and gathering information about students to assess what they are learning so as to ensure that every student is learning sound and significant mathematics and is developing a positive disposition toward mathematics;
- challenge and extend students' ideas;
- adapt or change activities while teaching;
- describe and comment on each student's learning to parents and administrators; and
- provide regular feedback to the students themselves.

STANDARD 7 Reflection on Teaching Practice

The teacher of mathematics should engage in ongoing analysis of teaching by—

- reflecting regularly on what and how they teach;
- examining effectiveness of the task, discourse, and learning environment on students' mathematical knowledge, skills, and dispositions;
- seeking to improve their teaching and practice by participating in learning communities beyond their classroom;
- analyzing and using assessment data to make reasoned decisions about necessary changes in curriculum; and
- collaborating with colleagues to develop plans to improve instructional programs.
Shift 1: From same instruction toward differentiated instruction.

Same instruction for all students. → Differentiated instruction, but same learning outcomes for all students.

Shift 2: From students working individually toward community of learners.

Students work individually on tasks and seek feedback from teacher on reasonableness of strategies and solutions. → Community of learners as part of which students hear, share, and judge reasonableness of strategies and solutions.

Shift 3: From mathematical authority coming from the teacher or textbook toward mathematical authority coming from sound student reasoning.

Correctness of solutions is determined by seeking input from teacher or textbook. → Correctness of solution is based on reasoning about the accuracy of the solution strategy.

Shift 6: From focus on correct answer toward focus on explanation and understanding.

Discussions and classroom routines focus on student explanation of how they solved a task and whether it is correct. → Discussions and classroom routines focus on student explanations addressing why an answer is (or isn’t) correct.

Facilitate meaningful mathematical discourse
Teacher and student actions

<table>
<thead>
<tr>
<th>What are teachers doing?</th>
<th>What are students doing?</th>
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<tbody>
<tr>
<td>Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations.</td>
<td>Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse.</td>
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<tr>
<td>Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion.</td>
<td>Listening carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments.</td>
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<tr>
<td>Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches.</td>
<td>Seeking to understand the approaches used by peers by asking clarifying questions, trying out others’ strategies, and describing the approaches used by others.</td>
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<tr>
<td>Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.</td>
<td>Identifying how different approaches to solving a task are the same and how they are different.</td>
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</table>
3b: Using Questioning and Discussion Techniques

The teacher uses a variety of questions or prompts to challenge students cognitively, advance high-level thinking and discourse, and promote metacognition. Students formulate many questions, initiate topics, challenge one another’s thinking, and make unsolicited contributions. Students themselves ensure that all voices are heard in the discussion.

CRITICAL ATTRIBUTES
- Students initiate higher-order questions.
- The teacher builds on and uses student responses to questions in order to deepen student understanding.
- Students extend the discussion, enriching it.
- Students invite comments from their classmates during a discussion and challenge one another’s thinking.
- Virtually all students are engaged in the discussion.

While the teacher may use some low-level questions, he poses questions designed to promote student thinking and understanding. The teacher creates a genuine discussion among students, providing adequate time for students to respond and stepping aside when doing so is appropriate. The teacher challenges students to justify their thinking and successfully engages most students in the discussion, employing a range of strategies to ensure that most students are heard.

CRITICAL ATTRIBUTES
- The teacher uses open-ended questions, inviting students to think and/or offer multiple possible answers.
- The teacher makes effective use of wait time.
- Discussions enable students to talk to one another without ongoing mediation by the teacher.
- The teacher calls on most students, even those who don’t initially volunteer.
- Many students actively engage in the discussion.
- The teacher asks students to justify their reasoning, and most students attempt to do so.

The teacher’s questions lead students through a single path of inquiry, with answers seemingly determined in advance. Alternatively, the teacher attempts to ask some questions designed to engage students in thinking, but only a few students are involved. The teacher attempts to engage all students in the discussion, to encourage them to respond to one another, and to explain their thinking, with uneven results.

CRITICAL ATTRIBUTES
- The teacher frames some questions designed to promote student thinking, but many have a single correct answer, and the teacher calls on students quickly.
- The teacher invites students to respond directly to one another’s ideas, but few students respond.
- The teacher calls on many students, but only a small number actually participate in the discussion.
- The teacher asks students to explain their reasoning, but only some students attempt to do so.

The teacher’s questions are of low cognitive challenge, with single correct responses, and are asked in rapid succession. Interaction between the teacher and students is predominantly recitation style, with the teacher mediating all questions and answers; the teacher accepts all contributions without asking students to explain their reasoning. Only a few students participate in the discussion.

CRITICAL ATTRIBUTES
- Questions are rapid-fire and convergent, with a single correct answer.
- Questions do not invite student thinking.
- All discussion is between the teacher and students; students are not invited to speak directly to one another.
- The teacher does not ask students to explain their thinking.
- Only a few students dominate the discussion.
### Exhibit 1—What Is and Is Not Student Mathematical Discourse

<table>
<thead>
<tr>
<th>IS Considered Discourse</th>
<th>IS NOT Considered Discourse</th>
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<tbody>
<tr>
<td>A student asks, “I don’t understand how you got that answer. Could you explain it again?”</td>
<td>The teacher provides an explanation of a mathematical procedure to the class.</td>
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<tr>
<td>A student explains, “I first added 20 and 40 to get 60. Then I subtracted 2 and added 3 to get 61.”</td>
<td>The teacher provides further explanation in response to a student’s question.</td>
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<tr>
<td>A student explains, “I saw that 18 + 43 was the same as (20 + 40) – 2 + 3.”</td>
<td>Two students discuss the scores of last week’s football game.</td>
</tr>
<tr>
<td>Students write in their journals about their thinking to solve a problem.</td>
<td>The teacher provides instructions to the class about an activity they are about to engage in.</td>
</tr>
<tr>
<td>A student states, “I think I see a pattern. Each one goes up by 3 more than the one before it.”</td>
<td>A student asks a question about nonmathematical procedures related to an assignment such as when the assignment is due, whether students need to show their work, and the like.</td>
</tr>
<tr>
<td>Two students discuss whether a procedure suggested by a student will work in all similar situations.</td>
<td>Students practice applying a procedure to solve problems of a specific type (seat work).</td>
</tr>
<tr>
<td>A student challenges an algorithm posed by a student by saying, “Yes, but how does it work with 37 x 98?”</td>
<td>The teacher provides a counter example to a method posed by a student.</td>
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<tr>
<td>A student answers a question in response to the teacher.</td>
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</table>

### Exhibit 3—Types of Mathematical Discourse

<table>
<thead>
<tr>
<th>Code</th>
<th>Level</th>
<th>Definition</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>Answering</td>
<td>A student gives a short answer to a direct question from the teacher or another student.</td>
</tr>
<tr>
<td>S</td>
<td>2</td>
<td>Making a Statement or Sharing</td>
<td>A student makes a simple statement or assertion, or shares his or her work with others and the statement or sharing does not involve an explanation of how or why. For example, a student reads what she wrote in her journal to the class.</td>
</tr>
<tr>
<td>E</td>
<td>3</td>
<td>Explaining</td>
<td>A student explains a mathematical idea or procedure by stating a description of what he or she did, or how he or she solved a problem, but the explanation does not provide any justification of the validity of the idea or procedure.</td>
</tr>
<tr>
<td>Q</td>
<td>4</td>
<td>Questioning</td>
<td>A student asks a question to clarify his or her understanding of a mathematical idea or procedure.</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>Challenging</td>
<td>A student makes a statement or asks a question in a way that challenges the validity of a mathematical idea or procedure. The statement may include a counter example. A challenge requires someone else to reevaluate his or her thinking.</td>
</tr>
<tr>
<td>R</td>
<td>6</td>
<td>Relating</td>
<td>A student makes a statement indicating that he or she has made a connection or sees a relationship to some prior knowledge or experience.</td>
</tr>
</tbody>
</table>
5.3 Planning Tool: Questioning across Lesson Phases

Questions vary with phases of a lesson. Here are some questions you might use with a teacher in a planning conversation. Discuss which questions might be appropriate to pose in an upcoming lesson.

### Launching the Task
- What is the task asking you to do?
- What do you already know about this topic?
- What information do you have? What do you need to find out?
- What strategies might you use to solve this problem?
- What diagram, visual, manipulative, or table might you use to solve the problem?
- What might your product (final solution) look like so that your classmates understand it?

### Monitoring the Task—As students work

#### One-on-One
- Where have you seen something like this before?
- What might happen if I changed this part of the problem?
- How is your strategy working?
- What might be another way to think about this problem?
- How might a simpler problem help you solve this problem?
- How might a tool such as a number line, picture, or manipulative help you?
- What patterns are you noticing?
- Does your answer seem reasonable? Why or why not?

**Question focused on mathematics of the lesson (objective):**

#### Small Group
Use one-on-one questions, plus the following:
- What do you think of [group member's] strategy?
- How are [two group members'] strategies alike or different?
- Explain how [group member] solved the task.
- How did you reach your conclusion(s)?
- What might be a more efficient strategy? Or which of the strategies in your group are efficient?
- Explain why you chose to organize your results this way.
- Will this work with other numbers? Explain.
- Are there other possibilities? How can you be sure?

**Question focused on mathematics of the lesson (objective):**

#### Whole Class
To monitor thinking as students are still working, use the following:
- What are some strategies you are using to solve the problem?
- What have you noticed about this problem?
- What do you think about what [student] said?
- Do you agree? Why or why not?
- Does anyone have the same answer but a different way to explain it?
- Do you understand what [student] is saying?
- Can you give me an example of . . . ?

**Question focused on mathematics of the lesson (objective):**

### Summarizing the Task—Whole Class
[To discuss task after students have solved it]

- How did you solve the problem?
- How might you convince the rest of us that your answer makes sense?
- Is that true for all cases or can you think of a counterexample?
- How does this relate to . . . ?
- What ideas that we have previously learned were useful in solving this problem?
- What would happen if . . . ? If _______ changes, how does it affect _______?
- What have you learned or found out today?
- What are the key points or big ideas in this lesson?
Figure 5.2  Productive Talk Moves

<table>
<thead>
<tr>
<th>Talk Moves</th>
<th>What It Means and Why</th>
<th>Example Teacher Prompts</th>
</tr>
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</table>
| 1. Revoicing | This move involves restating the statement as a question in order to clarify, to apply appropriate language, and to involve more students. It is an important strategy for reinforcing language and enhancing comprehension for ELLs. | “You used the hundreds chart and counted on?”  
“So—first you recorded your measurements in a table?” |
| 2. Rephrasing | Asking students to restate someone else’s ideas in their own words will ensure that ideas are stated in a variety of ways, as well as encouraging students to listen to each other. | “Who can share what Ricardo just said, but using your own words?” |
| 3. Reasoning | Rather than restate, as in talk move 2, this move asks the student what he or she thinks of an idea proposed by another student. | “Do you agree or disagree with Johanna? Why?” |
| 4. Elaborating | This is a request for students to challenge, add on, elaborate, or give an example. It is intended to increase student participation, deepen student understanding, and provide extensions. | “Can you give an example?”  
“Do you see a connection between Julio’s idea and Rhonda’s idea?”  
“What if...” |
| 5. Waiting | Ironically, one “talk move” is to not talk. Quiet time should not feel uncomfortable, but should feel like thinking time. If it gets awkward, ask students to pair-share and then try again. | “This question is important. Let’s take some time to think about it.” |


Table 1

<table>
<thead>
<tr>
<th>Levels</th>
<th>Characteristics of Discourse</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>The teacher asks questions and affirms the accuracy of answers or introduces and explains mathematical ideas. Students listen and give short answers to the teacher’s questions.</td>
</tr>
<tr>
<td>1</td>
<td>The teacher asks students direct questions about their thinking while other students listen. The teacher explains student strategies, filling in any gaps before continuing to present mathematical ideas. The teacher may ask one student to help another by showing how to do a problem.</td>
</tr>
<tr>
<td>2</td>
<td>The teacher asks open-ended questions to elicit student thinking and asks students to comment on one another’s work. Students answer the questions posed to them and voluntarily provide additional information about their thinking.</td>
</tr>
<tr>
<td>3</td>
<td>The teacher facilitates the discussion by encouraging students to ask questions of one another to clarify ideas. Ideas from the community build on one another as students thoroughly explain their thinking and listen to the explanations of others.</td>
</tr>
</tbody>
</table>

Adapted from Hufferd-Ackles, Fuson, and Sherin (2004)