

Ms. Rhee's math class was studying statistics. She brought in three bags containing red and blue marbles. The three bags were labeled as shown below:



Ms. Rhee shook each bag. She asked the class, "If you close your eyes, reach into a bag, and remove 1 marble, which bag would give you the best chance of picking a blue marble?"

Which bag would you choose?

Explain why this bag gives you the best chance of picking a blue marble. You may use the diagram above in your explanation.

Strategy	Who and What	Order

Taken from Smith, M. S., Hughes, E. K., Engle, R. A., & Stein, M. K. (2009). Orchestrating discussions. *Mathematics Teaching in the Middle School*, 14(9), 549-556.

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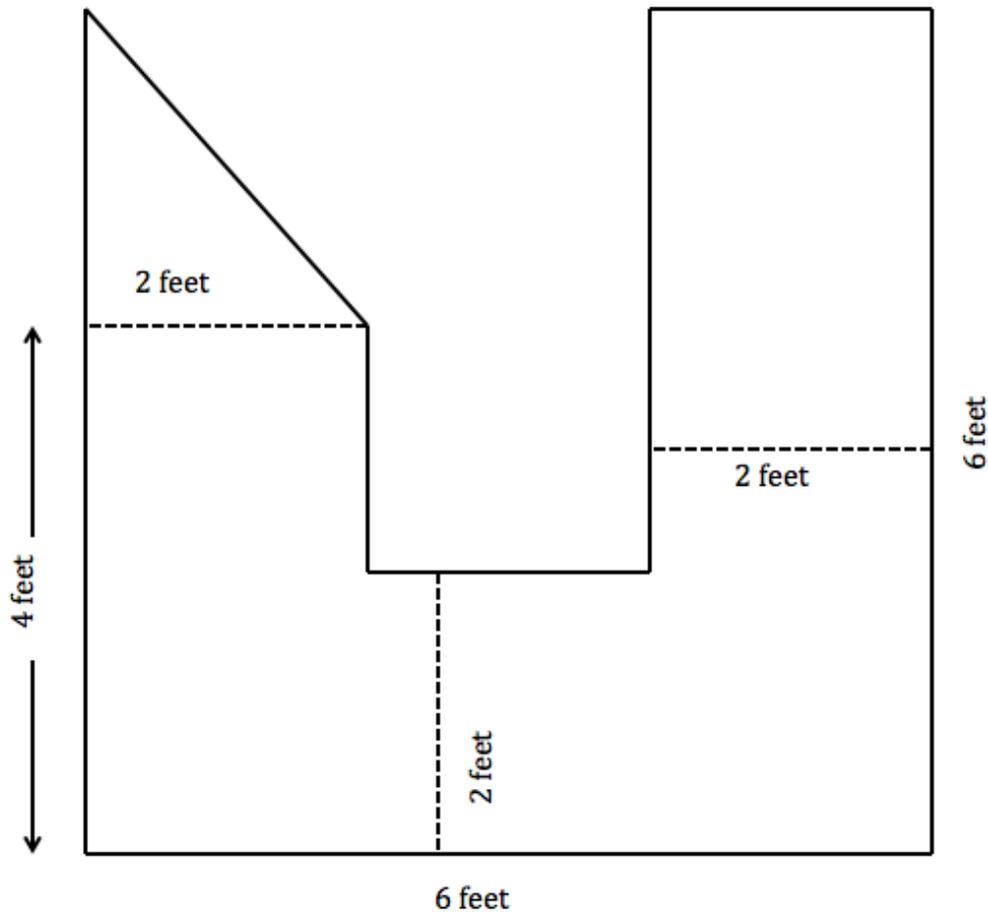
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<p>Fraction: Determine the fraction of each bag that is blue marbles (x is $\frac{1}{4}$; y is $\frac{1}{3}$; z is $\frac{1}{5}$). Decide which of the three fractions is larger ($\frac{1}{3}$). Select the bag with the largest fraction of blue marbles (bag y).</p>	<p><i>D</i>: Student does not explain <i>how</i> he or she determined that $\frac{1}{3}$ was the largest fraction.</p> <p><i>A</i>: Student created ratios but treats them as though they were fractions</p>	2nd
<p>Percent: Determine the fraction of each bag that is blue marbles (x is $\frac{25}{100}$; y is $\frac{20}{60}$; z is $\frac{25}{125}$). Change each fraction to a percent (x is 25 percent; y is $33\frac{1}{3}$ percent; z is 20 percent). Select the bag with the largest percent of blue marbles (bag y).</p>	<p><i>B</i>: Student does not explain which bag gives you the best chance and why.</p>	3rd
<p>Ratio (Unit Rate): Determine the part-to-part ratio that compares red with blue marbles for each bag (x is 3:1; y is 2:1; z is 4:1). Determine which bag has the fewest red marbles for every 1 blue marble (bag y)</p>	<p><i>C</i>: Student found the unit rates but did not use <i>this information</i> to answer the question.</p> <p><i>G</i>: Student compares blue to red ratio but does not explain why a 1:2 chance is better than the other</p>	4th
<p>Ratio (Scaling Up): Scale up each bag so that the number of blue marbles in each bag is the same (e.g., x is 300 R and 100 B; y is 200 R and 100 B; z is 400 R and 100 B). Select the bag that has the fewest red marbles for 100 blue marbles (bag y).</p>	<p>No one used this approach</p>	
<p>Additive: Determine the difference between the number of red and blue marbles in each bag (x is 50; y is 20; z is 75). Select the bag that has smallest difference (bag y).</p>	<p><i>F</i>: Student picked the right bag but for the wrong reason.</p>	First
<p>Other: Reasoning Combination of Fractions and Ratio</p>	<p><i>H</i>: Student compares <i>x</i> and <i>z</i> and concludes that <i>x</i> gives a better chance but does not consider bag <i>y</i>.</p> <p><i>E</i>: Student combines approaches seen in response D (compares the fractions $\frac{1}{4}$ and $\frac{1}{3}$), then switches to compare part-to-part ratios as in solution G.</p>	

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<p>Bag x is $\frac{1}{3}$ blue. Bag y is $\frac{1}{2}$ blue. Bag z is $\frac{1}{4}$ blue. $\frac{1}{2}$ is a lot so it must be bag y.</p> <p style="text-align: center;">A</p>	<p>I found the % of blue marbles in each bag. x: $\frac{25}{100} = 25\%$ y: $\frac{20}{60} = 33 \frac{1}{3}\%$ z: $\frac{25}{125} = 20\%$</p> <p style="text-align: center;">B</p>
<p>x: $\frac{75}{25} = 3/1 = 3$ y: $\frac{40}{20} = 2/1 = 2$ z: $\frac{100}{25} = 4/1 = 4$</p> <p>Since the marbles in bag z total 125, I think your chances would be higher than the others.</p> <p style="text-align: center;">C</p>	<p>Because bag y is $\frac{1}{3}$ full of blue marbles bag x is only $\frac{1}{4}$ full of blue marbles bag z is only $\frac{1}{5}$ full of blue marbles</p> <p style="text-align: center;">D</p>
<p>Bag x is $\frac{1}{4}$ blue and bag y is $\frac{1}{3}$ blue. Better chance bag y. Bag y has 1 blue to 2 reds, and bag z has 1 blue to 4 reds. Better chance bag y.</p> <p style="text-align: center;">E</p>	<p>The x bag has 75 red and 25 blue. There are 50 extra marbles that are red. The z bag has 100 red and 25 blue. There are 75 extra red than blue. Now Bag x has 40 red and 20 blue. There are 20 extra red than blue.</p> <p style="text-align: center;">F</p>
<p>Notice in the first bag there are 75 red and 25 blue, that is a 1:3 chance. Notice that in the second bag there are 40 red 20 blue, that is a 1:2 chance. Notice in the third bag there are 100 red 25 blue, that is a 1:4 of a chance. This shows that in bag y you would be likely to pick a blue marble.</p> <p style="text-align: center;">G</p>	<p>Bag x has 75 red and 25 blues and bag z has 100 red and 25 blues. In bags x and z the blues are the same, so then you would have to look at the red to see which is the least between them, and bag x has 75 red and 75 is less than 100, so I chose bag x.</p> <p style="text-align: center;">H</p>

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5.NF.A.1	For a school relay, four kids ran a total of 1 mile. If each kid ran the same distance, what fraction a mile did each kid run?	For a school relay, four kids ran a total of 1 mile. What fraction of a mile could each kid have run? Show two different ways using numbers and a visual fraction model.
6.RP.A.3	In a trail mix, the ratio of M&M's to pretzels is 2 cups to 3 cups. If you added 30 cups of M&M's, how many cups of pretzels must you add to the trail mix?	Write a story situation that uses the ratio 2 to 3.
6.RP.A.3	72 is what percent of 252?	Fill in the blank: 72 is ____% of ____
6.SP.A.5	Find the mean and median of the following set of data. 2, 6, 3, 8, 3, 6, 9, 3, 4	Create a data set with 10 data points where the mean of the data set is greater than the median.
7.NS.A.1	Find the sum: $-3 + 4 + -2$	Create sets of integers that will result in a sum of -1. Use a minimum of 3 integers in each set.
7.G.B.6	Find the volume and surface area of a right rectangular prism with the dimensions of 3 feet by 15 by 2 feet.	A prism has a surface area of 126 square inches. What could the dimensions be?
7.EE.A.1	Simplify the following expression: $2(x-3) + 4x - 4$	What could be an equivalent expression for $2(x-3) + 4x - 4$?
8.G.A.3	A triangle with vertices (1, 2), (5, 2), and (5, 7) is reflected across the x-axis. What are the vertices of the final triangle?	One vertex of a triangle is at (1,2). After a reflection one vertex of the triangle is at (5, 8). Name all three vertices of the original triangle and the final triangle.
8.EE.B.6	Find the slope of the line that passes through the points (2, -4) and (-7, 2).	A line has a slope of 2/3. Find three different sets of points that the line could pass through. Justify your reasoning.
HSA.SSE.B.3.B	Factor the quadratic expression: $2x^2 - x - 6$.	Given the quadratic expression $2x^2 + bx - 6$, what is a value of b that will make the quadratic expression factorable? What is a value of b that will make the quadratic expression not factorable?



Sabrina wants to replace her walkway with pavers sold in 1-foot squares. She wants to find the total area of her walkway. Three students solved the problem. Compare the students' work and explain who is correct.

$$\begin{aligned} (6)(4) &= 24 \text{ ft}^2 \\ (2)(2) &= 4 \text{ ft}^2 \\ \frac{1}{2}(2)(2) &= 2 \text{ ft}^2 \end{aligned}$$

$$24 + 4 - 2 = 26 \text{ ft}^2$$

$$\begin{aligned} (6)(2) &= 12 \text{ ft} \\ (4)(2) &= 8 \text{ ft} \\ \frac{1}{2}(6)(2) &= 6 \text{ ft} \end{aligned}$$

$$12 + 8 + 6 = 26 \text{ ft}$$

$$\begin{aligned} (6)(6) &= 36 \text{ ft}^2 \\ (4)(2) &= 8 \text{ ft}^2 \\ \frac{1}{2}(2)(2) &= 2 \text{ ft}^2 \end{aligned}$$

$$36 - 8 - 2 = 26 \text{ ft}^2$$