Title: Mastery of Angle Relationships Over Time

Description: We will demonstrate the progression of one learning target (angle relationships) from our Core Connections Geometry course (CPM) over time. Participants will do part of a lesson on angle relationships that our students do. They will experience some of our instructional strategies for group work, complete an angle relationships tool-kit, and review what they've learned in the homework. We will also explore our assessment program. Participants will take part of a formative assessment on angle relationships, learn how we help students in learning labs who did poorly on the formative assessment, take a summative assessment on angle relationships, and learn about how students become eligible for a retake summative assessment.

Speakers: Jenny Bergs  jbergs@deforestschools.org
          Erin Yao  ebalazs@deforestschools.org

Agenda:
Introductions
Study Teams, Classwork, and Huddle
Tool-Kit Entry
Review & Preview
Break/Questions
Formative Test
Learning Labs
Summative Test
Test Retake
Questions

CPM Mission
CPM Educational Program strives to make middle school and high school mathematics accessible to all students. It does so by collaborating with classroom teachers to create problem-based textbooks and to provide the professional development support necessary to implement them successfully.
Team Roles

General Team Roles

Resource Manager
Responsible for getting materials and asking the teacher questions.
- Get supplies for your team and make sure that your team cleans up.
- Help your team decide when it needs outside help. Assist in creating team questions for the teacher. Call the teacher over for team questions.

Facilitator
Responsible for keeping your team working together.
- Get your team started by having someone read the task out loud.
- Check that everyone understands what to work on.
- Make sure that each team member has shared his or her ideas.
- Make sure no one is getting left out or left behind. Make sure each person has time to write his or her answer before your team moves on.

Recorder/Reporter
Responsible for verifying that your team is writing justifications and explanations.
- Make sure that each team member can see the work your team is discussing.
- Make sure that your team agrees about how to explain and justify your answers, and that everyone understands your team’s answer.
- Make sure that each member of your team is able to share their ideas.

Task Manager
Responsible for facilitating an effective, participating team.
- Help keep your team on task, talking about math, and respecting each other’s right to learn.
- Keep track of time if you have been given a time limit and make sure your team is making progress at an appropriate pace.
- Make sure that no one talks outside your team.

Classwork

2.1.3 More Angles Formed by Transversals

These angle relationships are the ones students would have learned thus far:

<table>
<thead>
<tr>
<th>Straight Angles</th>
<th>Vertical Angles</th>
<th>Corresponding Angles</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Straight Angles Diagram" /></td>
<td><img src="image" alt="Vertical Angles Diagram" /></td>
<td><img src="image" alt="Corresponding Angles Diagram" /></td>
</tr>
</tbody>
</table>
2-25. Suppose $\angle a$ in the diagram below measures $48^\circ$.

![Diagram of two intersecting lines with angles labeled a and b.]

a. Use what you know about vertical, corresponding, and supplementary angle relationships to find the measure of $\angle b$.

b. Julia is still having trouble seeing the angle relationships clearly in this diagram. Her teammate, Althea, explains, "When I translate one of the angles along the transversal, I notice its image and the other given angle are a pair of vertical angles. That way, I know that angles $a$ and $b$ must be congruent."

Use Althea’s method and tracing paper to determine if the following angle pairs are congruent or supplementary. Be sure to state whether the pair of angles created after the translation is a vertical pair or forms a straight angle. Be ready to justify your answer for the class.

![Diagram with three pairs of angles labeled a, b, and c.]

2-26. ALTERNATE INTERIOR ANGLE RELATIONSHIP

In problem 2-25, Althea showed that the shaded angles in the diagram are congruent. However, these angles also have a name for their geometric relationship (their relative positions on the diagram). These angles are called alternate interior angles. They are called "alternate" because they are on opposite sides of the transversal, and "interior" because they are both inside (that is, between) the parallel lines.

![Diagram of two parallel lines with transversal cutting through them.]

a. Find another pair of alternate interior angles in this diagram.

b. Think about the relationship between the measures of alternate interior angles. If the lines are parallel, are they always congruent? Are they always supplementary? Complete the conjecture, "If lines are parallel, then alternate interior angles are..."

c. Instead of writing conditional statements, Rosie likes to write arrow diagrams to express her conjectures. She expresses the conjecture from part (b) as:

\text{Lines are parallel $\rightarrow$ alternate interior angles are congruent.}

This arrow diagram says the same thing as the conditional statement you wrote in part (b). How is it different from your conditional statement? What does the arrow mean?

d. Prove that alternate interior angles are congruent. That is, how can you use rigid transformations to move $\angle CFG$ so that it lands on $\angle BCP$? Explain. Be sure your team agrees.
**Huddle**

If you want to check on the progress of the teams, give additional information or ask for a consensus on something to do with the problem everyone is working on, you could call a “huddle.” Have one person from each team come to a meeting (at the front of the room or just outside the door). You give that person the information and they return to the team and share it. This is a great way to give status to a low status team member. It gives them something important to share with the team. It is also a way to communicate with all the teams without having to call the whole class together.

Find more study team strategies here: [http://www.cpm.org/pdfs/studyTeam/STTS%20cards%202012.pdf](http://www.cpm.org/pdfs/studyTeam/STTS%20cards%202012.pdf)

---

**Tool-Kit Entry**

Lesson 2.1.3 Resource Page

### Angle Relationships Toolkit

In the space below, describe what you know about these geometric angle relationships. Be sure to include what you know about the relationship of their angle measures (such as are they ever supplementary? If so, when?). Include a diagram.

<table>
<thead>
<tr>
<th>Vertical Angles</th>
<th>Straight Angles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Corresponding Angles</th>
<th>Alternate Interior Angles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Same-Side Interior Angles</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Review & Preview

Note that these angle relationships problems show up in multiple chapters – mastery over time!

2-66. Examine the diagram below. Then use the information provided in the diagram to find the measures of angles $a$, $b$, $c$, and $d$. For each angle, name the relationship from your Angle Relationships Toolkit that helped justify your conclusion. For example, did you use vertical angles? If not, what type of angle did you use? [Homework Help]

![Diagram](image)

CL 2-122. Identify the geometric angle relationship(s) in each diagram. Use what you know about those relationships to write an equation and solve for $x$.

a.

![Diagram](image)

b.

![Diagram](image)

c.

![Diagram](image)

3-22. Examine the relationships in the diagram below. Then solve for $x$ and $y$, if possible. Justify your work using angle relationships. [Homework Help]

![Diagram](image)

8-108. Find the missing angle(s) in each problem below using the geometric relationships shown in the diagram below. Be sure to write down the conjecture that justifies each calculation. Remember that each part is a separate problem. [Homework Help]

![Diagram](image)

a. If $d = 110^\circ$ and $b = 5x - 20^\circ$, write an equation and solve for $x$.

b. If $b = 4x - 11^\circ$ and $n = x + 20^\circ$, write an equation and solve for $x$. Then find the measure of $\angle n$. 
Formative Tests

At the end of each chapter, we give students two tests. The formative test is on new material they just learned in that chapter. The summative test is on material from previous chapters. We score formative and summative tests by learning targets. Each learning target gets a score of 0 or 5-10.

Chapter 2 Formative Test

1. For each diagram calculate the value of x. Show your work and include an explanation of what you used (definitions and theorems) to solve the problem.
   a. 
   b. 

   ![Diagram](image1.png)

2. Identify the kinds of angles in each diagram. Write a complete conjecture that describes the type of angle and their relationship with one another (congruent or supplementary).
   a. 
   b. 
   c. 

   ![Diagram](image2.png)
Learning Labs

We have math learning labs built into our schedule every 3 days in advisory for 30 minutes and every 6 days in study hall for 70 minutes. Geometry students can sign up for either a Geometry Target Lab or Geometry Homework Lab during advisory. In the Geometry Target Lab, we review one or two learning targets that will be on their next summative test. In the Geometry Homework Lab, students can receive help on their Review & Preview assignments. During Geometry Study Hall, students can get extra help on their Review & Preview, get extra help and practice on specific learning targets, and/or work on retaking tests.

DAHS Schedule with Learning Labs

Sample Geometry Target Learning Lab Schedule (Posted in All Geometry Classrooms)

<table>
<thead>
<tr>
<th>Date</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/17/14</td>
<td>Area and Perimeter (Triangles, Parallelograms, Trapezoids)</td>
</tr>
<tr>
<td>10/22/14</td>
<td>Multiple Representations of Functions</td>
</tr>
<tr>
<td>10/28/14</td>
<td>Angle Relationships</td>
</tr>
<tr>
<td>10/31/14</td>
<td>Probability</td>
</tr>
<tr>
<td>11/5/14</td>
<td>Pythagorean Theorem and Triangle Inequality Theorem</td>
</tr>
</tbody>
</table>

The CPM website had study guides for most of the learning targets we assess in Geometry. We use these study guides for the Geometry Target Learning Labs in advisory.

http://www.cpm.org/students/extraByCourseCCG.htm
Summative Tests

Only learning targets that have been previously tested formatively show up on the summative test.

Chapter 3 Summative Test

1. For each of the following problems solve for \( x \).

   a. 
   ![Diagram](image1.png)
   b. 
   ![Diagram](image2.png)

2. **Identify** the kinds of angles in each diagram. Write a complete conjecture that names the relationship and describes the relationship.

   a. 
   ![Diagram](image3.png)
   b. 
   ![Diagram](image4.png)
Test Retakes

Test Retake Policy: Students may retake any chapter test, but must make a remediation plan with the teacher which may include some or all of the following:

- Students meet with teacher to go over the original test and make corrections.
- Students must show that homework is completed for that chapter.
- Students complete additional practice problems until teacher sees that they have mastered the learning targets.
- Students attend learning labs.

After students have successfully demonstrated that they have mastered the learning targets, students are eligible to retake that test for full credit. This process typically takes 3 to 5 visits (outside of class) before it is completed.

References:
www.cpm.org
http://commons.wikimedia.org/wiki/File:Angle_straight.png#/media/File:Angle_straight.png
http://upload.wikimedia.org/wikipedia/commons/1/13/Vertical_Angles.svg

Thank you for coming to our session! We hope you learned something helpful to bring back to your classroom. Please don’t hesitate to email us if you have any questions.