MS TASKS TO BUILD PROFICIENCY IN PROBABILITY AND STATISTICS

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MSP: MATH TRANSITION INTO THE COMMON CORE ERA

- Rice Lake Primary LEA Partner
- UW-River Falls, UW-Eau Claire, IHE Partners
- Summer Academies 2014 MS Focus
  - Probability and Statistics
  - Math Practices
15 PARTNER SCHOOL DISTRICTS

- Rice Lake
- Barron
- Cameron
- Lake Holcombe
- Luck
- Osseo-Fairchild
- Eau Claire

- Chippewa Falls
- Baldwin Woodville
- Ellsworth
- River Falls
- Menomonie
- Prairie Farm
- Prescott
- Bruce
Rich tasks

- Our group worked to find and modify tasks to implement in the 2014-2015 school year

Explore websites with rich tasks

- Classroom tested activities and assessments
- Enhance current curricular resources
RICH TASKS

- Multiple representations
- Open-ended problems
- Incorporate the Math Practices
- Exploration for deeper learning, connections to prior knowledge
MULTIPLE REPRESENTATIONS - PROBABILITY OF COMPOUND EVENTS

- Coin: 2/8 = 25%
- Spinner: 2/8 = 25%
- Possible Outcomes:
  - H: 2
  - R: 1
  - Y: 2
  - G: 3
- Table:
  - H: 1
  - R: 1
  - Y: 1
  - G: 5
- Probability of Compound Events:
  - **H** × **R** = 1/4
PRACTICE STANDARDS

- MP1: Make sense of problems and persevere in solving them.
- MP2: Reason abstractly and quantitatively.
- MP3: Construct viable arguments and critique the reasoning of others.
- MP4: Model with mathematics.
- MP5: Use appropriate tools strategically.
- MP6: Attend to precision.
- MP7: Look for and make use of structure.
- MP8: Look for and express regularity in repeated reasoning.
WEBSITES

- Illustrative Mathematics
- MARS – Mathematics Assessment Project
- Yummy Math
- Illuminations
- Engage NY
- Inside Mathematics
TAKE AWAY FOR THE CLASSROOM: 
SETTING EXPECTATIONS

Illustrative Mathematics Activity

1. You will work with your table partner today.

2. Complete the Illustrative Mathematics Problem.

3. Reminder -
   a. Be persistent
   b. Communicate your work
   c. Model your answers when appropriate.

Source: Olson & Munden, 2013
7.SP Rolling Twice

Task

A fair six-sided die is rolled twice. What is the theoretical probability that the first number that comes up is greater than or equal to the second number?

Extension – Find your experimental probability.

Alignments to Content Standards: 7.SP.C.8
Solution: 1 Plotting outcomes in a table

We can plot the different possible outcomes as the six sided die is rolled. One way of doing this is with a table as shown below.

<table>
<thead>
<tr>
<th>1,1*</th>
<th>1,2</th>
<th>1,3</th>
<th>1,4</th>
<th>1,5</th>
<th>1,6</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,1*</td>
<td>2,2*</td>
<td>2,3</td>
<td>2,4</td>
<td>2,5</td>
<td>2,6</td>
</tr>
<tr>
<td>3,1*</td>
<td>3,2*</td>
<td>3,3*</td>
<td>3,4</td>
<td>3,5</td>
<td>3,6</td>
</tr>
<tr>
<td>4,1*</td>
<td>4,2*</td>
<td>4,3*</td>
<td>4,4*</td>
<td>4,5</td>
<td>4,6</td>
</tr>
<tr>
<td>5,1*</td>
<td>5,2*</td>
<td>5,3*</td>
<td>5,4*</td>
<td>5,5*</td>
<td>5,6</td>
</tr>
<tr>
<td>6,1*</td>
<td>6,2*</td>
<td>6,3*</td>
<td>6,4*</td>
<td>6,5*</td>
<td>6,6*</td>
</tr>
</tbody>
</table>

In the table the entry marked 1,3 means that the first throw was a 1 and the second throw a 3. An asterisk next to the entry means that the first number that came up is greater than or equal to the second number. There are 36 possibilities listed in the table, each equally likely. For the 21 starred cases, the first number was greater than or equal to the second number. So the probability that the first number is greater than or equal to the second number is \( \frac{21}{36} = \frac{7}{12} \).

Solution: 2 Plotting outcomes in a list

If the first number is a 1, this can only be bigger than or equal to the second number if the second number is a 1. If the first number is a 2, then this is bigger than or equal to 1 or 2. We can make a table for all possibilities that the first number is greater than or equal to the second:

<table>
<thead>
<tr>
<th>Number on first throw</th>
<th>Possible numbers on second throw</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1,2</td>
</tr>
<tr>
<td>3</td>
<td>1,2,3</td>
</tr>
<tr>
<td>4</td>
<td>1,2,3,4</td>
</tr>
<tr>
<td>5</td>
<td>1,2,3,4,5</td>
</tr>
<tr>
<td>6</td>
<td>1,2,3,4,5,6</td>
</tr>
</tbody>
</table>

Adding up the possibilities in the table, there are 21 ways that the number on the second roll can be greater than or equal to the number on the first roll. Since there are \( 6 \times 6 = 36 \) total possible outcomes, the probability that the first number is greater than or equal to the second is \( \frac{21}{36} = \frac{7}{12} \).
Solution: 3 Abstract analysis of outcomes

There are 6 outcomes for the first roll and 6 for the second so the total number of possible outcomes is $6 \times 6 = 36$. There are 6 of these in which the two numbers are equal: (1,1), (2,2), (3,3), (4,4), (5,5), and (6,6). When the two numbers are not equal, half of these cases have a greater first number and half have a greater second number: this can be seen by reversing the order to the two throws, which exchanges these two cases. So of the 30 cases where the two numbers are different, 15 of them have a greater first number. This means the first number is greater than or equal to the second in $6 + 15 = 21$ of the 36 different possible outcomes. So the probability that the first throw is at least as big as the second is $\frac{21}{36} = \frac{7}{12}$.
STANDARDS FOR TASK

CCSS.MATH.CONTENT.7.SP.C.8
Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

CCSS.MATH.CONTENT.7.SP.C.8.A
Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

CCSS.MATH.CONTENT.7.SP.C.8.B
Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

CCSS.MATH.CONTENT.7.SP.C.8.C
Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?
This task is designed to practice mean, median, and mode.

It expects students to explain and justify their answers.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>Rubric</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>points</td>
</tr>
<tr>
<td>1.a</td>
<td>Table completed correctly.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer: total $680,000</td>
<td></td>
</tr>
<tr>
<td>1.b</td>
<td>Gives correct answer: $45,333</td>
<td></td>
</tr>
<tr>
<td></td>
<td>and shows calculation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$80,000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$80,000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$80,000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$680,000</td>
<td></td>
</tr>
<tr>
<td>2.a</td>
<td>Gives correct explanation such as: He has not looked at how many people earn each salary</td>
<td></td>
</tr>
<tr>
<td>2.b</td>
<td>Gives correct answer: $30,000</td>
<td></td>
</tr>
<tr>
<td>3.a</td>
<td>Gives correct answer: $40,000</td>
<td></td>
</tr>
<tr>
<td>3.b</td>
<td>There are 15 people. The middle person, the 8th person, gets $40,000</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>Gives correct answer: Mean</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct explanation such as: That is the highest of the three.</td>
<td></td>
</tr>
<tr>
<td>5.a</td>
<td>Gives correct answer: Mode</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total Points</td>
</tr>
</tbody>
</table>
The MARS site also includes sample student responses. It includes samples that have been scored that can be discussed with students after they have completed the task. These can be used to teach students how to clearly show work and explain their reasoning.
Summarize and describe distributions.

4: Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

5: Summarize numerical data sets in relation to their context, such as by:
   a: Reporting the number of observations.
   b: Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
   c: Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
   d: Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
7th Grade Task – Dice Games from Yummy Math

- Dice are commonly used in probability models.
- There are dice and spinner simulators available online. For ex: http://www.shodor.org/interactivate/activities/ExpProbability/
- The 7th grade group are implementing the probability task from: http://www.yummymath.com/2014/games-with-dice-2/
- This task introduces experimental and theoretical probability by rolling dice.
- The group created a pre and post test with a rubric.
CCSS.MATH.CONTENT.7.SP.C.5
Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

CCSS.MATH.CONTENT.7.SP.C.6
Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.

CCSS.MATH.CONTENT.7.SP.C.7
Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
8TH GRADE - SAMPLE SMARTER BALANCED TASK

- Taken from the smarter balanced sample items at: http://sbac.portal.airast.org/practice-test/
- Includes the classroom activity to use prior to completing the task.
- Modified the computer version of the task to a paper version.
- Task involves drawing a line of fit and creating and analyzing an equation.
- After students completed the task on paper we had them go on to the smarter balanced practice test and complete it online.
1. The data from the table are plotted below. Draw a line of fit for the data.

2. Write an equation for the line of fit you drew in #1.

\[
\frac{-50}{300} = -0.17 = m \quad b = 120
\]

\[
y = -0.17x + 120
\]

* m between -.25 + -.05
* b between 70 + 130
3. Interpret the slope of the line in the context of the situation.

   The slope shows the pulse rate drops by about 0.17 beats/min for each kilogram added to the weight.

4. Based on your equation from #2, predict the average pulse rate in beats per minute, of an animal that weighs 6000 kilograms.

   \[-0.17(6000) + 120\]

   \[-900 \text{ beats/min}\]

5. Explain whether the predicted average pulse rate in #4 is reasonable in context of the situation.

   The value is not reasonable. You can't have a negative heart rate.
6. The body weight and pulse rate of a guinea pig and rabbit are given in the table below.

<table>
<thead>
<tr>
<th>Animal</th>
<th>Average Body Weight (in kg)</th>
<th>Average Pulse Rate (in beats per minute)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guinea Pig</td>
<td>1</td>
<td>250</td>
</tr>
<tr>
<td>Rabbit</td>
<td>2.5</td>
<td>265</td>
</tr>
</tbody>
</table>

If the study included these data, would this change the model relating average body weight and average pulse rate? How do you know?

The new data points would not fit the trend described by the line of fit or the equation from #2. The model would need to change to fit these new points.
8TH GRADE – BARBIE BUNGEES

- **Source:** [http://illuminations.nctm.org/Lesson.aspx?id=2157](http://illuminations.nctm.org/Lesson.aspx?id=2157)

- Students collect data to create a scatter plot.

- They then create a line of fit to determine a safe number of rubber bands for Barbie to jump a larger distance.

- Responses are then tested by having Barbie make a jump at 400 cm.
VIDEO
1. Complete the data table below.

<table>
<thead>
<tr>
<th>Number of Rubber Bands (x)</th>
<th>Jump Distance in Centimeters (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>58.7 cm</td>
</tr>
<tr>
<td>4</td>
<td>94 cm</td>
</tr>
<tr>
<td>6</td>
<td>134 cm</td>
</tr>
<tr>
<td>8</td>
<td>152 cm</td>
</tr>
<tr>
<td>10</td>
<td>180 cm</td>
</tr>
<tr>
<td>12</td>
<td>200 cm</td>
</tr>
</tbody>
</table>

2. Make a scatterplot of your data. Indicate the scale on each axis.
5. What is the equation for your line of best fit? (You may wish to use a graphing calculator for this part of the lesson. Enter the rubber band data in L1, and enter the jump distance data for L2.)

\[ y = 2.40x + 23.2 \]

6. What is the slope of your equation, and what does it represent in this context?

\[ m = 2.40 = \text{number of rubber bands} \]

7. What is the y-intercept of your equation, and what does it represent in this context?

23.2 = jump distance

8. Based on your data, what would you predict is the maximum number of rubber bands so that Barbie could still safely jump from 400 cm?

Using your Line of Best Fit: 23

Using your Regression Equation: 24

The correct solution seemed to be around 28 rubber bands.
During our summer session the 8th grade group modified a task to implement this school year.

The task involved interpreting two-way tables.

“Lesson 10: Summarizing Bivariate Categorical Data with Relative Frequencies”

Task source: engageNY site - Algebra I - Module 2 - Lesson 10

The group created a pre and post test with a rubric.
CCSS.MATH.CONTENT.8.SPA.1
Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

CCSS.MATH.CONTENT.8.SPA.2
Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
CCSS.MATH.CONTENT.8.SP.A.3
Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*

CCSS.MATH.CONTENT.8.SP.A.4
Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*
During our summer session we completed a box and whisker plot activity.

Each person walked down the hallway with their eyes covered.

The distance the person was able to walk between two pieces of tape was recorded.

A box and whisker plot was then created on the floor to represent the data.

It is a great visual.
Summarize and describe distributions.

CCSS.MATH.CONTENT.6.SP.B.4

Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
THANK YOU – QUESTIONS?

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