OVERVIEW

• What is basic fact fluency?
• Games to support basic fact fluency
• Assessing basic facts: The facts
• Assessment strategies for basic fact fluency
What does fluency with basic facts mean?
CCSS-M Descriptions

Grade 1 (1.0A.C.6):

Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known sums.

Grade 2 (2.0A.B.2):

Fluently add and subtract within 20 using mental strategies (reference to 1.0A.C.6). By end of Grade 2, know from memory all sums of two one-digit numbers.
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Grade 3 (3.0A.C.7):

Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
CCSS-M Descriptions

Grade 3 (3.0A.C.7):

_Fluently_ multiply and divide within 100, _using strategies_ such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, _know from memory_ all products of two one-digit numbers.
This would suggest that *fluency is different* from automatic retrieval. Research heavily supports this…

So, what does fluency *really* mean?
The Common Core State Standards for Mathematics (CCSS-M) describes procedural fluency as “skill in carrying out procedures flexibly, accurately, efficiently and appropriately” (CCSSO, 2010, p. 6).
Procedural Fluency

Knowing from Memory ≠ Memorization

Outcome

Strategy
Developing Fact Fluency
Mastering Basic Facts

Phase 1: Counting
(counts with objects or mentally)

Phase 2: Deriving
(uses reasoning strategies based on known facts)

Phase 3: Mastery
(efficient production of answers)

Adapted from Baroody, 2006
Mastering Basic Facts: Addition

Phase 1 Example: Solving $5 + 7$ by counting on from 5.

Phase 2 Example: Solving $5 + 7$ by starting from $5 + 5 = 10$ and then adding 2 more to get 12.

Phase 3 Example: Answering $5 + 7 = 12$ with little/no hesitation.
# Reasoning Strategies for Addition Facts

## K-2 Trajectory

<table>
<thead>
<tr>
<th>Foundational Fact Strategies</th>
<th>Derived Fact Strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Sums within 5</td>
<td>• Near Doubles</td>
</tr>
<tr>
<td>• +/- 1 or 2</td>
<td>1 2</td>
</tr>
<tr>
<td>• Doubles</td>
<td>(6 + 7, 8 + 7)</td>
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<tr>
<td>(2 + 2, 6 + 6, etc.)</td>
<td>• Making Ten</td>
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<tr>
<td>• Combinations of Ten</td>
<td>1 2</td>
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<tr>
<td>(3 + 7, 8 + 2)</td>
<td>(8 + 3, 9 + 5)</td>
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</tbody>
</table>

*Note: K1 indicates a strategy for Kindergarten 1.*
## Addition Fact Fluency: A 3-year Progression

<table>
<thead>
<tr>
<th>K</th>
<th>1</th>
<th>2</th>
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<tbody>
<tr>
<td>Sums within 5</td>
<td>Doubles</td>
<td>Near Doubles</td>
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<tr>
<td>+/- 1</td>
<td>Combo’s of 10</td>
<td>Making 10</td>
</tr>
<tr>
<td>+/- 2</td>
<td>Sums within 10</td>
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<tr>
<td>Combo’s of 10</td>
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</tbody>
</table>
Mastering Basic Facts: Multiplication

Phase 1 Example: Solving $6 \times 4$ by drawing 6 groups of 4 dots and skip counting the dots.

Phase 2 Example: Solving $6 \times 4$ by thinking $5 \times 4 = 20$ and adding one more group of 4.

Phase 3 Example: Knowing that $6 \times 4 = 24$ (or doing Phase 2 really fast)
# Reasoning Strategies for Multiplication Facts

## Foundational Fact Strategies
- 2s, 5s, 10s
- 0’s*, 1’s
- Multiplication squares

## Derived Fact Strategies
- Adding or subtracting a group
- Halving and doubling
- Nearby square
- Decomposing a factor
Multiplication Fact Fluency: A Third Grade Priority

Products within 5 as repeated addition

- Foundational Fact Strategies
- Derived Fact Strategies
- Fluency
Meaningful Activities for Learning Basic Facts

- Solve Number Stories
- Use Quick Looks with dot patterns and ten frames
- Discuss/write about strategy use
- Play basic facts games with a focus on reasoning strategies
Story Telling

(a) Join

Initial -> Change -> Result

(b) Separate

Initial -> Change -> Result

(c) Part-part-whole

Part | Part
--- | ---
Whole

(d) Compare

Large set

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<thead>
<tr>
<th>Difference</th>
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</table>

Small set
Phase 1 ➔ Phase 2

Quick Images

PreK:
✓ number recognition
✓ representation

Kindergarten:
✓ decomposing and recomposing numbers
✓ subitizing

Grade K-2:
✓ recognizing and developing strategies for basic +/- combinations
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Developing Fact Strategies through Quick Looks

- How can Quick Looks reinforce learning of foundational facts?
- What arrangements on double ten frames can encourage fact strategies such as near doubles and making ten?
- Why is the Quick Look format important to eliciting these strategies?
“Practice that follows substantial initial experiences that support understanding and emphasize ‘thinking strategies’ has been shown to improve student achievement with single-digit calculations.” (NRC, 2001).
Where does each game fit in?

Post it!

Phase 1: Counting

Phase 2: Deriving

Phase 3: Mastery

Game
Games Across Phases

1. Tens Go Fish
2. Close to 20
3. Double It
4. Top-It
5. Tetris
6. Strive to Derive
Game #1: Tens Go Fish

Play *Go Fish*, a match is a combination that makes 10 (instead of a match).
Where does each game fit in?
Post it!

Phase 1: Counting

Phase 2: Deriving

Phase 3: Mastery

Game
Game #2: Close to 20

1. Use deck of cards with numbers 0–10 (Ace = 1, Jack = 0)
2. Turn over 5 cards.
3. Pick 3 cards that have a sum as close to 20 as possible.
4. Score is difference from 20.
5. Play 5 rounds. (low score wins)
Where does the game fit in?

Post it!

Phase 1: Counting

Phase 2: Deriving

Phase 3: Mastery

Game
Game #3: Double It!

1. Flip a card (or roll a die).
2. Fill in the Double Fact on the table.
3. Optional: First to have 5 tall wins.

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<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>16</th>
<th>18</th>
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</tbody>
</table>
Where does the game fit in?

Post it!

Phase 1: Counting

Phase 2: Deriving

Phase 3: Mastery

Game
Game #4: Top It!

Work in pairs.
1. Each person draws 2 cards.
2. Say the sum or product.
3. Check partners answer.
4. Largest total keeps the cards (like war).
5. Play 5 rounds.
Where does the game fit in?
Post it!

Phase 1: Counting

Phase 2: Deriving

Phase 3: Mastery

Game
Game #5

MULTIPLICATION TETRIS
Game #5: TETRIS

Whole Group
1. Create a 10-by-10 grid
2. Shade each dice pair and write product.
3. Stay in the game longer than anyone else!
Where does the game fit in?
Post it!

Phase 1: Counting

Phase 2: Deriving

Phase 3: Mastery

Game
Game #6: Strive to Derive

Work in pairs
1. Each person draws two cards (rolls two dice). [prepare 1 die with only 3s, 6s, 9s, the other with selected numbers]
2. Find the array card (sketch it).
3. Partition to show 2 or 5 fact.
4. Say the product of the parts and the answer.
Game #6: Strive to Derive

Work in pairs
1. Each person draws two cards (rolls two dice). [prepare 1 die with only 3s, 6s, 9s; the other with selected numbers]
2. Find the array card (sketch it).
3. Partition to show 2 or 5 fact.
4. Say the product of the parts and the answer.
Work in pairs

1. Each person draws two cards (rolls two dice). [prepare 1 die with only 3s, 6s, 9s; the other with selected numbers]

2. Find the array card (sketch it).

3. Partition to show 2 or 5 fact.

4. Say the product of the parts and the answer.
Where does the game fit in?

Post it!

Phase 1: Counting

Phase 2: Deriving

Phase 3: Mastery

Game
Meaningful Practice

Games:

• Are engaging.
• Provide opportunities for strategy discussion and assessment.
• Should be sequenced developmentally (for example, playing combinations of ten games before exploring making ten strategies).
• Can be targeted practice or general practice.
• Lend to differentiation.
November 2014 article includes ADDITION games
May 2015 TCM article includes multiplication games
NEW Books with Basic Facts Games (NCTM)

Addition  Multiplication
Assessing Fact Fluency
The Common Core State Standards for Mathematics (CCSS-M) describes procedural fluency as “skill in carrying out procedures flexibly, accurately, efficiently and appropriately” (CCSSO, 2010, p. 6).
Assessing Basic Fact Fluency

What can we learn from assessments related to:

- Flexibility
- Accuracy
- Efficiency
- Appropriate Strategy Use
Aspects of Fluency

- Flexibility
- Accuracy
- Efficiency
- Appropriate Strategy Use

Timed Tests

(1) 19 + 1 =  (10) 2 + 4 =  (19) 10 + 2 =  (28) 1 + 3 =
(2) 9 + 4 =  (11) 9 + 1 =  (20) 5 + 3 =  (29) 5 + 4 =
(3) 5 + 4 =  (12) 15 + 1 =  (21) 14 + 3 =  (30) 2 + 2 =
(4) 8 + 4 =  (13) 16 + 1 =  (22) 5 + 3 =  (31) 11 + 1 =
(5) 14 + 4 =  (14) 16 + 1 =  (23) 7 + 2 =  (32) 14 + 1 =
(6) 15 + 3 =  (15) 8 + 4 =  (24) 5 + 1 =  (33) 7 + 4 =
(7) 14 + 4 =  (16) 15 + 2 =  (25) 9 + 2 =  (34) 6 + 4 =
(8) 15 + 1 =  (17) 9 + 1 =  (26) 2 + 1 =  (35) 4 + 2 =
(9) 14 + 2 =  (18) 15 + 4 =  (27) 12 + 1 =  (36) 10 + 1 =
Timed Testing: Issues

The issues with timed testing include:

1) Limitations as an assessment tool
2) Can impede progress when mastering facts
3) Psychological effects
Timed Testing: Issues

1) Limitations with respect to the four components of fluency.

A child finishes a 20-fact timed test in 60 seconds.

- Did the child spend 3 seconds on each fact?
  
  Or...
  
- Did the child spend 1 second on 16 facts and 10 seconds each on 4 of the facts?
Timed Testing: Issues

2) Can impede progress in mastering facts

A study of 2\textsuperscript{nd} and 4\textsuperscript{th} graders showed that children in experimental classrooms with a focus on strategy development vastly outperformed those in the control classrooms, even on traditional timed assessments.

Thornton, 1978
Timed Testing: Issues

2) Can impede progress in mastering facts

A study of nearly 300 first graders found that children who were more frequently exposed to timed testing demonstrated lower progress towards knowing facts from memory than their counterparts.

Henry & Brown, 2008
Timed Testing: Issues

3) Can have negative psychological effects

- The stress that children experience with timed testing is not experienced when they complete the same tasks in untimed conditions.
- “Evidence strongly suggests that timed tests cause the early onset of math anxiety for students across the achievement range.”

Boaler, 2014
3) Can have negative psychological effects

Anxiety over timed testing is often not related to achievement. Even high-achieving children share concerns such as “I feel nervous. I know my facts, but this just scares me.”

Boaler, 2012
Timed Testing: Issues

3) Can have negative psychological effects

Children experience math anxiety as early as first grade and this anxiety is not correlated with reading achievement. This suggests that the children’s anxiety is specific to mathematics, not general academic work.

Ramirez et al. 2013
Timed Testing: Issues

3) Can have negative psychological effects

Children who tended to use more sophisticated mathematical strategies experienced the most negative impact on achievement due to math anxiety. Thus, it appears that some of our best mathematical thinkers are often those most negatively impacted by timed testing.

Ramirez et al. 2013
Timed Tests: Alternatives

- Observation
- Strategy quizzes
- Self-assessment
- Writing prompts
- Interviews
Observation Checklist

Addition Facts Fluency Chart

<table>
<thead>
<tr>
<th>Student</th>
<th>Models and counts all</th>
<th>Counts on Fact</th>
<th>Derived Fact</th>
<th>Recall (double or combo of 10)</th>
<th>Recall</th>
<th>Comments</th>
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Aspects of Fluency

- Flexibility
- Accuracy
- Efficiency
- Appropriate Strategy Use
Salute!
Aspects of Fluency

- Flexibility
- Accuracy
- Efficiency
- Appropriate Strategy Use

Journal Writing

If your friend didn’t know the answer to $4 + 5$, how would you tell him to figure it out?
Journal Writing

Review the four student responses:

What might you infer about each child’s level of fluency?
May 10, 2012
I would tell my friend
to take 5 and
count 4 in your hand
I would tell my friend to start with 5 then add 2 then one more 2 and then you have 9.
I would tell my friend to "ooos a double plus 1. 4 + 4 = 8. so count up. now you get your answer.}
I would tell my friend
to take away one number from ten.
And that is nine.
I know that five plus five equals ten.
## Interview Questions

### Flexibility

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
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</thead>
<tbody>
<tr>
<td>Solve 6 + 7 using one strategy. Now try solving it using a different strategy.</td>
<td>What is the answer to 7 + 8? How do you know it is correct (how might you check it)?</td>
</tr>
</tbody>
</table>

### Accuracy

<table>
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### Efficiency

<table>
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<tr>
<th>Question</th>
<th>Answer</th>
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<tr>
<td>For which facts did you just know?</td>
<td>Emily solved 6 + 8 by changing it in her mind to 4 + 10. What did she do? Is this a good strategy? Tell why or why not.</td>
</tr>
<tr>
<td>For which facts did you use a strategy?</td>
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</table>

### Appropriate Strategy Selection

<table>
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Tests: Alternatives

Addition Fact Fluency Quiz

Solve these problems and tell how you solved out.

4 + 5 = _____  Check one:  ____ I used this strategy: ____________
                  ____ I just knew.

10 + 6 = _____  Check one:  ____ I used this strategy: ____________
                  ____ I just knew.

6 + 2 = _____   Check one:  ____ I used this strategy: ____________
                  ____ I just knew.
Assessing Basic Fact Fluency

Have you had it with timed tests, which present a number of concerns and limitations? Try a variety of alternative assessments from this sampling that allows teachers to accurately and appropriately measure children’s fact fluency.

By Gina Kling and Jennifer M. Bay-Williams
What Works with Addition Fact Fluency
Promising Results

Kling (2013) followed 30 children from 2 different schools, 4 different classrooms, in Kalamazoo, MI.; 21 had no exposure to timed testing or drill in the classroom in either 1st or 2nd grade. By the end of 2nd grade those 21 children demonstrated:

- automaticity with addition facts (solved within 3 seconds) 95% of the time.
- Strategy use (e.g., making ten) so quickly that it was impossible to distinguish between strategy use and “knowing from memory.”
... and RETENTION

18 of the children were interviewed once more in the first week of 3rd grade prior to any fact strategy review.

- Children demonstrated automaticity 91% of the time.
- Were accurate and used strategies (not counting) 99.99% of the time.
Conclusions

Basic facts instruction, practice, and assessment must truly encompass all four components of fluency:
- **Flexibility**
- **Accuracy**
- **Efficiency**
- **Appropriate strategy use**

Traditional approaches to teaching and assessing basic facts do not support these goals. But when fluency is the focus, children can achieve meaningful mastery of basic facts.
Grade 2 (2.0A.B.2):

Fluently add and subtract within 20 using mental strategies (reference to 1.0A.C.6). By end of Grade 2, know from memory all sums of two one-digit numbers.

Grade 3 (3.0A.C.7):

Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
Assessing Your Fact Fluency
Whip Around

Share your answer to one of these prompts:

• An activity I will use is...
• An assessment strategy I will use is...
• Something surprising I heard is...
• I am going to/not going to...
Thank you! You are great – that’s a FACT
j.baywilliams@louisville.edu
Bibliography


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