Using Visual Models to Build Ratio and Proportional Reasoning

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For each pair of tennis shoes Jayla owns, she owns 4 pairs of flip flops. If Jayla has 9 more pairs of flip flops than tennis shoes, how many pairs of flip flops does she have? How many pairs of tennis shoes?
Tape Diagram

Jayla has 12 pairs of flip flops and 3 pairs of tennis shoes.

Double Number Line

Jayla has 12 pairs of flip flops and 3 pairs of tennis shoes.

Table

<table>
<thead>
<tr>
<th>Tennis shoes</th>
<th>Flip Flops</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>9</td>
</tr>
</tbody>
</table>

Jayla has 12 pairs of flip flops and 3 pairs of tennis shoes.
What is the importance of using a visual model when solving math problems?
Teaching Practice 3

Use and connect mathematical representations.

"Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving."

NCTM: Principles to Actions
### Use and connect mathematical representations

<table>
<thead>
<tr>
<th>What are teachers doing?</th>
<th>What are students doing?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selecting tasks that allow students to decide which representations to use in making sense of the problems.</td>
<td>Using multiple forms of representations to make sense of and understand mathematics.</td>
</tr>
<tr>
<td>Allocating substantial instructional time for students to use, discuss, and make connections among representations.</td>
<td>Describing and justifying their mathematical understanding and reasoning with drawings, diagrams, and other representations.</td>
</tr>
<tr>
<td>Introducing forms of representations that can be useful to students.</td>
<td>Making choices about which forms of representations to use as tools for solving problems.</td>
</tr>
<tr>
<td>Asking students to make math drawings or use other visual supports to explain and justify their reasoning.</td>
<td>Sketching diagrams to make sense of problem situations.</td>
</tr>
<tr>
<td>Focusing students’ attention on the structure or essential features of mathematical ideas that appear, regardless of the representation.</td>
<td>Contextualizing mathematical ideas by connecting them to real-world situations.</td>
</tr>
<tr>
<td>Designing ways to elicit and assess students’ abilities to use representations meaningfully to solve problems.</td>
<td>Considering the advantages or suitability of using various representations when solving problems.</td>
</tr>
</tbody>
</table>

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_NCTM: Principles to Actions_
Teacher Actions: What are teachers doing?

Categorize them

As I think about my teaching...
(a) I really haven't given much thought or attention to this topic.
(b) I need to learn more about this topic.
(c) I try, but I need to place more emphasis on this topic.
(d) I consciously include this topic into my teaching.

*NCTM: Principles to Actions*
Student Actions: What are students doing?

Categorize them

As I think about myself as a student...
(a) I really haven't given much thought or attention to that topic.
(b) I need to learn more about this topic.
(c) I try, but I need to place more emphasis on this topic.
(d) I consciously include this topic into my learning.

NCTM: Principles to Actions
p.24 “When students learn to represent, discuss, and make connections among mathematical ideas in multiple forms, they demonstrate deeper mathematical understanding and enhanced problem-solving abilities.”

p. 25 “Visual representations are of particular importance in mathematics classrooms, helping students to advance their understanding of mathematical concepts and procedures, make sense of problems, and engage in mathematical discourse. Visuals support problem solving as students consider relationships among quantities and when they sketch diagrams or make tables and graphs. The visual representations also support discourse because the diagrams or drawings leave a trace of student problem solving that can be displayed, critiqued, and discussed.”
For every seven chores that Brandon completes, his sister completes five. Last month, Brandon completed 24 more chores than his sister. How many chores did his sister complete last month?
What can you tell about students' understanding of proportional relationships from looking at their models?
The Alden Middle School girls’ soccer team won 80% of their games this season. If the team won 12 games, how many games did they play?
Tape Diagram

The Alden Middle School girls’ soccer team won 80% of their games this season. If the team won 12 games, how many games did they play?

\[12 \div 4 = 3\]
\[3 \times 5 = 15\]

The soccer team played 15 games.

Double Number Line

The Alden Middle School girls’ soccer team won 80% of their games this season. If the team won 12 games, how many games did they play?

The soccer team played 15 games.

Table

The Alden Middle School girls’ soccer team won 80% of their games this season. If the team won 12 games, how many games did they play?

<table>
<thead>
<tr>
<th>Percent</th>
<th>Number of Games</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>12</td>
</tr>
<tr>
<td>20</td>
<td>3</td>
</tr>
<tr>
<td>100</td>
<td>15</td>
</tr>
</tbody>
</table>

The soccer team played 15 games.
Last week, the ratio of boys to girls who visited the doctor was 2:3. This week the ratio of boys to girls who visited the doctor was 1:4. If the same number of kids visited the doctor both weeks and 51 girls visited the doctor last week, how many boys and how many girls visited the doctor this week?
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This week 17 boys and 68 girls went to the doctor.

Table

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<table>
<thead>
<tr>
<th>Boys</th>
<th>Girls</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>14</td>
<td>21</td>
<td>35</td>
</tr>
<tr>
<td>20</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td>34</td>
<td>51</td>
<td>85</td>
</tr>
</tbody>
</table>

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<tr>
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<th>Total</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>5</td>
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<tr>
<td>17</td>
<td>68</td>
<td>85</td>
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</tbody>
</table>

This week 17 boys and 68 girls went to the doctor.
The ratio of bluebirds to blackbirds is 2:5. If there are 56 birds in the sky total, how many of these birds are blue?
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\[
\begin{align*}
\text{Blue} & \quad \frac{16}{8} \\
\text{Black} & \quad 8 \quad 8 \quad 8 \quad 8 \quad 8
\end{align*}
\]

\[
\frac{56}{7} = 8
\]

There are 16 blue birds.
A furnace used 40% of the fuel in its tank in March, and then used 25% of the remaining fuel in April. If there were 240 gallons at the beginning of March, how much fuel was left at the end of April?
Double Number Line

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\[
\begin{align*}
\text{March} & \\
0 & \rightarrow 0 \rightarrow 60 \rightarrow 144 \rightarrow 240 \\
40\% & \rightarrow 60\% \\
240 & \times 6 = 144 \\
100\% - 40\% & = 60\% \\
144 & \div 4 = 36 \\
36 & \times 3 = 108
\end{align*}
\]

There were 108 gallons in the tank at the end of April.

Tape Diagram

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\[
\begin{align*}
\text{March} & \\
48 & \rightarrow 48 \rightarrow 48 \rightarrow 48 \rightarrow 48 \\
240 & \div 5 = 48 \\
48 & \times 3 = 144
\end{align*}
\]

\[
\begin{align*}
\text{April} & \\
36 & \rightarrow 36 \rightarrow 36 \rightarrow 36 \rightarrow 144 \\
144 & \div 4 = 36 \\
36 & \times 3 = 108
\end{align*}
\]

There were 108 gallons in the tank at the end of April.
Harry's amount of money is 75% of Kayla's amount of money. After Harry earned $30 and Kayla earned 25% more of her money, Harry's amount of money is 80% of Kayla's money. How much money did each have at the beginning?
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![Diagram]

Originally, the number line had 4 segments. Because of the added money, another 25% was added. The number line then has 5 segments. Each segment is worth $30. $30 · 3 = Harry's amount. $30 · 4 = Kayla's amount.

Therefore, Harry started with $90 and Kayla started with $120.
Additional Resources

