Introducing parametric equations

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Lesson Outline

Note: I am setting the stage to show that while \( y \) may not be a function of \( x \), both \( x \) and \( y \) can be functions of a third variable (i.e. parameter)

1) Opening Question

Consider an ant that walks aimlessly around a coordinate plane. Can its path be described using a function? Why or why not? (Not guaranteed since the ant could have multiple \( y \)-positions for a single \( x \)-position.)

2) Follow up question

When tracking one’s path, are there any other variables that must be considered? (yes, time)

3) Lead in to parametrics

Suppose we treat the horizontal and vertical positions independently of each other.

Is the \( y \)-position a function of time? (Yes, we can have only one vertical position at a given time?)

Is the \( x \)-position a function of time? (Yes, we can have only one horizontal position at a given time?)

4) Conclusion.

As long as we view \( x \)- and \( y \)- independent of each other and have them controlled by a third variable, we can link \( x \) and \( y \) when describing curves.

5) Begin discussion of applications

Can you think of a situation where we control horizontal and vertical movements simultaneously but yet independently to draw out curves?

(Depending on student experiences, they may or may not be able to think of situations. With prodding (“I suspect many of you had such a device as a young child” etc.) invariably somebody will come up with the child’s toy, Etch-A-Sketch. After the usual commentary of “I could only draw rectangles” and the like, I inform them such is not the case in the hands of a master.)

Show Etch-A-Sketch video

- Disney Frozen: [https://www.youtube.com/watch?v=xTdTX2e4dJY](https://www.youtube.com/watch?v=xTdTX2e4dJY)
- Chicago cubs: [https://www.youtube.com/watch?v=UyCe3RNT7pw](https://www.youtube.com/watch?v=UyCe3RNT7pw)

Manufacturing uses: CNC

- CNC instructional video: [https://www.youtube.com/watch?v=mMEnk5I4DWE](https://www.youtube.com/watch?v=mMEnk5I4DWE)
- CNC cutout: [https://www.youtube.com/watch?v=ZWh6ch_JAT0](https://www.youtube.com/watch?v=ZWh6ch_JAT0)
- Water jet cutting glass: [https://www.youtube.com/watch?v=Q_Aq90MIE_c](https://www.youtube.com/watch?v=Q_Aq90MIE_c)

6) Complete worksheet
A CNC (computer numerical control) router is being used to carve out the letters in a stone plaque. The following set of parametric equations describes the vertical and horizontal movement of the router as it carves out a single upper-case letter. Both $x$ and $y$ are in centimeters and $t$ is in seconds. Can you predict which letter it will carve?

a) How many seconds does it take to carve the letter? _________

b) Over each of the indicated time intervals, describe the movement of the router through the material.

<table>
<thead>
<tr>
<th>Interval</th>
<th>Horizontal and vertical movement of router</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq t \leq 2$</td>
<td></td>
</tr>
<tr>
<td>$2 \leq t \leq 5$</td>
<td></td>
</tr>
<tr>
<td>$5 \leq t \leq 7$</td>
<td></td>
</tr>
</tbody>
</table>

c) Graph the $(x, y)$ path of the router on the axes below. Identify the time values associated with each key location.
2) Predict which letter is created by the following set of parametric equations? _____ Graph the movement in the xy-plane. Identify the router’s location for each integer time value. Note: Dotted lines are movement without carving.

3) Consider the upper-case ‘A’ shown below. Using the directed paths and indicated time values graph the individual $y$ (vertical) and $x$ (horizontal) movement of the router over time.

4) For a final assessment on this activity, you are going to create a parametric “pattern” for a letter of your choosing. (No, you cannot use a letter already used within this activity.)
   a) On the leftmost axes below, draw an upper-case letter indicating the router movements and time values similar to that in the preceding problem. The letter must not exceed 4 units wide or 6 units tall.
   b) Draw the vertical and horizontal movements over time on the associated axes.