NUMBER TALKS AT-A-GLANCE

1. THINK
   → Say and write the expression on board (horizontally)
   → Wait until most students have a thumb up (a total)

2. LISTEN/SHARE
   → Call on 4-5 students to share answers only; write answers on the board
   → Students use “same” signal if they had the same total
   → Accept all answers (even incorrect ones) without saying if they are correct
   → Ask: can both/all these answers be correct? (this isn’t an everyday step, just once in awhile as a reminder that there can only be one correct answer for each equation)

3. EXPLAIN/DEFEND
   → Select a student to share his/her solution to the equation
   → Chart student thinking on board—try to chart exactly what students say, even if they are incorrect; give them opportunities to correct/clarify their own thinking before jumping in to “save” them
   → Take time to name the strategy used (i.e. counting on, making a ten, using friendly numbers)
   → Students use “same” signal if they had the same total
   → Repeat the process with another student’s strategy

4. QUESTION [this may come later with younger students, after they have grown more comfortable with the Number Talks routine]
   → Allow students to question each other about their thinking or the strategy they chose
   → Have students identify similarities/differences between strategies

Silent Signals
- READY → closed fist on chest
- I HAVE AN ANSWER → put thumb up
- I HAVE ANOTHER STRATEGY → put out a finger for each additional strategy
- SAME THINKING → move hand back and forth to show agreement

WMC 2015: Number Talks: Changing the Culture of Math
**COMMON ADDITION STRATEGIES**
1. Counting All
2. Counting On
3. Doubles/Near Doubles
4. Making Tens
5. Making Landmark/Friendly Numbers
6. Compensation
7. Breaking into Place Value
8. Adding Up in Chunks
(Parrish, 59-66)

**COMMON SUBTRACTION STRATEGIES**
1. Counting Back
2. Adding Up
3. Removal
4. Using Place Value and Negative Numbers
5. Adjusting One Number to Make an Easier Problem
6. Keeping a Constant Difference
(Parrish, 171-180)

**COMMON MULTIPLICATION STRATEGIES**
1. Repeated Addition or Skip Counting
2. Making Landmark or Friendly Numbers
3. Partial Products
4. Doubling and Halving
5. Breaking Factors into Smaller Factors
(Parrish, 245-252)

**COMMON DIVISION STRATEGIES**
1. Repeated Subtraction
2. Multiplying Up
3. Partial Quotients
4. Proportional Reasoning
(Parrish, 254-260)

**RESOURCES:**
- Number Talks K-5: [http://schoolwires.henry.k12.ga.us/Page/37070](http://schoolwires.henry.k12.ga.us/Page/37070)
- Number Talk Video: [https://mathsolutions.wistia.com/medias/0v5002j2zh](https://mathsolutions.wistia.com/medias/0v5002j2zh)

**Middle School Number Talk Resources**
- [http://www.svmimac.org/images/Cristo_Rey_-_Middle_Level_Bank.pdf](http://www.svmimac.org/images/Cristo_Rey_-_Middle_Level_Bank.pdf)
- [http://www.mathtalks.net/teachers.html](http://www.mathtalks.net/teachers.html)
Math Talk vs. Number Talks...They are NOT the Same!

Math Talk (Learning Conversations)
- “A respectful but engaged conversation in which students can clarify their own thinking and learn from others through talk” (Chapin, p 5)
- Should be used within the daily lesson in all areas of math study
- Can be used in all academic areas
- Students discuss a concept, procedure, solution method, idea, or definition in order to understand more deeply and with greater clarity.
- Teacher uses 5 Talk Moves to move along the conversation.

5 Productive Talk Moves for Math Talk
- Revoicing: Teacher repeats some or all of what a student said.
- Repeating: Teacher asks someone to restate another student’s comment.
- Reasoning: Teacher asks someone to apply their reasoning to someone else’s reasoning.
- Adding On: Teacher asks students to add new thoughts to the conversation.
- Waiting: Teacher uses wait time.

Math Talk Example
Some students are engage in a discussion about the relationship between squares and rectangles.
Amanda: Squares have two sets of parallel sides, and we already said rectangles do, too. So squares and rectangles are the same.
Teacher: Luis, do you agree? Why or why not?
Luis: I’m not sure, because they have the parallel sides, but squares don’t have long and short sides...but the angles are right angles.

Number Talks (Mental Math Computation)
- “Classroom conversations around purposefully crafted computation problems that are solved mentally” (Parrish, p xviii)
- Happen separately from the daily lesson
- May or may not connect to the lesson
- Last for 5 to 15 minutes
- Is quick paced
- Teacher poses problems, listens and charts students’ strategies.
- Students share strategies and try out new strategies.

Number Talks Example
Students solve the following problems in succession in order to develop the “Making Friendly Numbers” strategy.
- 99 + 5
- 99 + 17
- 99 + 26
During the third problem, students share:
Ramiro: Oh, I can take 1 from 26 and give it to 99. That’s 100 plus 25, which is 125.
Teacher charts: 99 + 66 = 100 + 25 = 125
Ramiro: I used 1 from the 26 to make that 100. Now the 26 is 1 less. It’s only 25.
Teacher: Amaya, did you use the same strategy?
Amaya: No, I know 100 + 26 is 126. Then I just had to subtract 1. It’s 125.
Teacher charts: 100 + 26 = 126
126 – 1 = 125
Teacher: You subtracted 1?
Amaya: Yes, because I added too much at first. I added 100 instead of 99.

While Math Talk refers to a way to structure discourse about any given topic, a Number Talk is a mini-lesson that supports computational fluency. A teacher may use some aspects of Math Talk, such as the talk moves, during a Number Talk, but Math Talks should be more fully employed during the main daily math lesson.

Sources
Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts

By Jo Boaler
Professor of Mathematics Education, co-founder youcubed

with the help of Cathy Williams, co-founder youcubed, & Amanda Confer
Stanford University.

Introduction

A few years ago a British politician, Stephen Byers, made a harmless error in an interview. The right honorable minister was asked to give the answer to 7 x 8 and he gave the answer of 54, instead of the correct 56. His error prompted widespread ridicule in the national media, accompanied by calls for a stronger emphasis on ‘times table’ memorization in schools. This past September the Conservative education minister for England, a man with no education experience, insisted that all students in England memorize all their times tables up to 12 x 12 by the age of 9. This requirement has now been placed into the UK’s mathematics curriculum and will result, I predict, in rising levels of math anxiety and students turning away from mathematics in record numbers. The US is moving in the opposite direction, as the new Common Core State Standards (CCSS) de-emphasize the rote memorization of math facts. Unfortunately misinterpretations of the meaning of the word 'fluency' in the CCSS are commonplace and publishers continue to emphasize rote memorization, encouraging the persistence of damaging classroom practices across the United States.

Mathematics facts are important but the memorization of math facts through times table repetition, practice and timed testing is unnecessary and damaging. The English minister’s mistake when he was asked 7 x 8 prompted calls for more memorization. This was ironic as his mistake revealed the limitations of memorization without ‘number sense’. People with number sense are those who can use numbers flexibly. When asked to solve 7 x 8 someone with number sense may have memorized 56 but they would also be able to work out that 7 x 7 is 49 and then add 7 to make 56, or they may work out ten 7’s and subtract two 7’s (70-14). They would not have to rely on a distant memory. Math facts, themselves, are a small part of mathematics and they are best learned through the use of numbers in different ways and situations. Unfortunately many classrooms focus on math facts in unproductive ways, giving students the impression that math facts are the essence of mathematics, and, even worse that the fast recall of math facts is what it means to be a strong mathematics student. Both of these ideas are wrong and it is critical that we remove them from classrooms, as they play a large role in the production of math anxious and disaffected students.

It is useful to hold some math facts in memory. I don’t stop and think about the answer to 8 plus 4, because I know that math fact. But I learned math facts through using them in different mathematical situations, not by practicing them and being tested on them. I grew up in the progressive era of England, when primary schools focused on the ‘whole child’ and I was not presented with tables of addition, subtraction or multiplication facts to memorize in school. This has never held me back at any time or place in my life,
even though I am a mathematics education professor. That is because I have number sense, something that is much more important for students to learn, and that includes learning of math facts along with deep understanding of numbers and the ways they relate to each other.

**Number Sense**

In a critical research project researchers studied students as they solved number problems (Gray & Tall, 1994). The students, aged 7 to 13, had been nominated by their teachers as being low, middle or high achieving. The researchers found an important difference between the low and high achieving students - the high achieving students used number sense, the low achieving students did not. The high achievers approached problems such as 19 + 7 by changing the problem into, for example, 20 + 6. No students who had been nominated as low achieving used number sense. When the low achieving students were given subtraction problems such as 21-16 they counted backwards, starting at 21 and counting down, which is extremely difficult to do. The high achieving students used strategies such as changing the numbers into 20 -15 which is much easier to do. The researchers concluded that low achievers are often low achievers not because they know less but because they don't use numbers flexibly – they have been set on the wrong path, often from an early age, of trying to memorize methods instead of interacting with numbers flexibly (Boaler, 2009). This incorrect pathway means that they are often learning a harder mathematics and sadly, they often face a lifetime of mathematics problems.

Number sense is the foundation for all higher-level mathematics (Feikes & Schwingendorf, 2008). When students fail algebra it is often because they don’t have number sense. When students work on rich mathematics problems – such as those we provide at the end of this paper - they develop number sense and they also learn and can remember math facts. When students focus on memorizing times tables they often memorize facts without number sense, which means they are very limited in what they can do and are prone to making errors –such as the one that led to nationwide ridicule for the British politician. Lack of number sense has led to more catastrophic errors, such as the Hubble Telescope missing the stars it was intended to photograph in space. The telescope was looking for stars in a certain cluster but failed due to someone making an arithmetic error in the programming of the telescope (LA Times, 1990). Number sense, critically important to students’ mathematical development, is inhibited by over-emphasis on the memorization of math facts in classrooms and homes. The more we emphasize memorization to students the less willing they become to think about numbers and their relations and to use and develop number sense (Boaler, 2009).

**The Brain and Number Sense**

Some students are not as good at memorizing math facts as others. That is something to be celebrated, it is part of the wonderful diversity of life and people. Imagine how dull and uninspiring it would be if teachers gave tests of math facts and everyone answered them in the same way and at the same speed as though they were all robots. In a recent brain study scientists examined students’ brains as they were taught to memorize math facts. They saw that some students memorized them much more easily than others. This will be no surprise to readers and many of us would probably assume that those who memorized better were higher achieving or “more intelligent” students. But the researchers found that the students who mem-
orized more easily were not higher achieving, they did not have what the researchers described as more “math ability”, nor did they have higher IQ scores (Supekar et al, 2013). The only differences the researchers found were in a brain region called the hippocampus, which is the area of the brain that is responsible for memorized facts (Supekar et al, 2013). Some students will be slower when memorizing but they still have exceptional mathematics potential. Math facts are a very small part of mathematics but unfortunately students who don’t memorize math facts well often come to believe that they can never be successful with math and turn away from the subject.

Teachers across the US and the UK ask students to memorize multiplication facts, and sometimes addition and subtraction facts too, usually because curriculum standards have specified that students need to be “fluent with numbers”. Parish, drawing from Fosnot and Dolk (2001) defines fluency as ‘knowing how a number can be composed and decomposed and using that information to be flexible and efficient with solving problems.’ (Parish 2014, p 159). Whether or not we believe that fluency requires more than the recall of math facts, research evidence points in one direction: The best way to develop fluency with numbers is to develop number sense and to work with numbers in different ways, not to blindly memorize without number sense.

When teachers emphasize the memorization of facts, and give tests to measure number facts students suffer in two important ways. For about one third of students the onset of timed testing is the beginning of math anxiety (Boaler, 2014). Sian Beilock and her colleagues have studied people’s brains through MRI imaging and found that math facts are held in the working memory section of the brain. But when students are stressed, such as when they are taking math questions under time pressure, the working memory becomes blocked and students cannot access math facts they know (Beilock, 2011; Ramirez, et al, 2013). As students realize they cannot perform well on timed tests they start to develop anxiety and their mathematical confidence erodes. The blocking of the working memory and associated anxiety particularly occurs among higher achieving students and girls. Conservative estimates suggest that at least a third of students experience extreme stress around timed tests, and these are not the students who are of a particular achievement group, or economic background. When we put students through this anxiety provoking experience we lose students from mathematics.

Math anxiety has now been recorded in students as young as 5 years old (Ramirez, et al, 2013) and timed tests are a major cause of this debilitating, often life-long condition. But there is a second equally important reason that timed tests should not be used – they prompt many students to turn away from mathematics. In my classes at Stanford University, I experience many math traumatized undergraduates, even though they are among the highest achieving students in the country. When I ask them what has happened to lead to their math aversion many of the students talk about timed tests in second or third grade as a major turning point for them when they decided that math was not for them. Some of the students, especially women, talk about the need to understand deeply, which is a very worthwhile goal, and being made to feel that deep understanding was not valued or offered when timed tests became a part of math class. They may have been doing other more valuable work in their mathematics classes, focusing on sense making and understanding, but timed tests evoke such strong emotions that students can come to believe that being fast with math facts is the essence of mathematics. This is extremely unfortunate. We see the outcome of the misguided school emphasis on memorization and testing in the numbers dropping out of mathematics and the math crisis we currently face (see www.youcubed.org). When my own daughter started times table
memorization and testing at age 5 in England she started to come home and cry about maths. This is not the emotion we want students to associate with mathematics and as long as we keep putting students under pressure to recall facts at speed we will not erase the widespread anxiety and dislike of mathematics that pervades the US and UK (Silva & White, 2013; National Numeracy, 2014).

In recent years brain researchers have found that the students who are most successful with number problems are those who are using different brain pathways – one that is numerical and symbolic and the other that involves more intuitive and spatial reasoning (Park & Brannon, 2013). At the end of this paper we give many activities that encourage visual understanding of number facts, to enable important brain connections. Additionally brain researchers have studied students learning math facts in two ways – through strategies or memorization. They found that the two approaches (strategies or memorization) involve two distinct pathways in the brain and that both pathways are perfectly good for life long use. Importantly the study also found that those who learned through strategies achieved 'superior performance' over those who memorized, they solved problems at the same speed, and showed better transfer to new problems. The brain researchers concluded that automaticity should be reached through understanding of numerical relations, achieved through thinking about number strategies (Delazer et al, 2005).

Why is Mathematics Treated Differently?

In order to learn to be a good English student, to read and understand novels, or poetry, students need to have memorized the meanings of many words. But no English student would say or think that learning about English is about the fast memorization and fast recall of words. This is because we learn words by using them in many different situations – talking, reading, and writing. English teachers do not give students hundreds of words to memorize and then test them under timed conditions. All subjects require the memorization of some facts, but mathematics is the only subject in which teachers believe they should be tested under timed conditions. Why do we treat mathematics in this way?

Mathematics already has a huge image problem. Students rarely cry about other subjects, nor do they believe that other subjects are all about memorization or speed. The use of teaching and parenting practices that emphasize the memorization of math facts is a large part of the reason that students disconnect from math. Many people will argue that math is different from other subjects and it just has to be that way – that math is all about getting correct answers, not interpretation or meaning. This is another misconception. The core of mathematics is reasoning - thinking through why methods make sense and talking about reasons for the use of different methods (Boaler, 2013). Math facts are a small part of mathematics and probably the least interesting part at that. Conrad Wolfram, of Wolfram-Alpha, one of the world's leading mathematics companies, speaks publically about the breadth of mathematics and the need to stop seeing mathematics as calculating. Neither Wolfram nor I are arguing that schools should not teach calculating, but the balance needs to change, and students need to learn calculating through number sense, as well as spend more time on the under-developed but critical parts of mathematics such as problem solving and reasoning.

It is important when teaching students number sense and number facts never to emphasize speed. In fact this is true for all mathematics. There is a common and damaging misconception in mathematics – the idea that strong math students are fast math students. I work with a lot of mathematicians and one thing I
notice about them is that they are not particularly fast with numbers, in fact some of them are rather slow. This is not a bad thing, they are slow because they think deeply and carefully about mathematics. Laurent Schwartz, a top mathematician, wrote an autobiography about his school days and how he was made to feel “stupid” because he was one of the slowest math thinkers in his class (Schwartz, 2001). It took him many years of feeling inadequate to come to the conclusion that: ‘rapidity doesn’t have a precise relation to intelligence. What is important is to deeply understand things and their relations to each other. This is where intelligence lies. The fact of being quick or slow isn’t really relevant.’ (Schwartz, 2001) Sadly speed and test driven math classrooms lead many students who are slow and deep thinkers, like Schwartz, to believe that they cannot be good at math.

**Math ‘Fluency’ and the Curriculum**

In the US the new Common Core curriculum includes ‘fluency’ as a goal. Fluency comes about when students develop number sense, when they are mathematically confident because they understand numbers. Unfortunately the word fluency is often misinterpreted. ‘Engage New York’ is a curriculum that is becoming increasingly popular in the US that has incorrectly interpreted fluency in the following ways:

*Fluency: Students are expected to have speed and accuracy with simple calculations; teachers structure class time and/or homework time for students to memorize, through repetition, core functions such as multiplication tables so that they are more able to understand and manipulate more complex functions.* (Engage New York)

There are many problems with this directive. Speed and memorization are two directions that we urgently need to move away from, not towards. Just as problematically ‘Engage New York’ links the memorization of number facts to students’ understanding of more complex functions, which is not supported by research evidence. What research tells us is that students understand more complex functions when they have number sense and deep understanding of numerical principles, not blind memorization or fast recall (Boaler, 2009). I am currently working with PISA analysts at the OECD. The PISA team not only issues international mathematics tests every 4 years they collect data on students’ mathematical strategies. Their data from 13 million 15-year olds across the world show that the lowest achieving students are those who focus on memorization and who believe that memorizing is important when studying for mathematics (Boaler & Zoido, in press). This idea starts early in classrooms and is one we need to eradicate. The highest achievers in the world are those who focus on big ideas in mathematics, and connections between ideas. Students develop a connected view of mathematics when they work on mathematics conceptually and blind memorization is replaced by sense making.

In the UK directives have similar potential for harm. The new national curriculum states that all students should have ‘memorised their multiplication tables up to and including the 12 multiplication table’ by the age of 9 and whilst students can memorize multiplication facts to 12 x 12 through rich engaging activities this directive is leading teachers to give multiplication tables to students to memorize and then be tested on. A leading group in the UK, led by children’s author and poet Michael Rosen, has formed to highlight the damage of current policies in schools and the numbers of primary age children who now walk to school crying from the stress they are under, caused by over-testing (Garner, The Independent, 2014). Mathematics is the leading cause of students’ anxiety and fear and the unnecessary focus on memorized math facts in the early years is one of the main reasons for this.
Activities to Develop Number Facts and Number Sense

Teachers should help students develop math facts, not by emphasizing facts for the sake of facts or using ‘timed tests’ but by encouraging students to use, work with and explore numbers. As students work on meaningful number activities they will commit math facts to heart at the same time as understanding numbers and math. They will enjoy and learn important mathematics rather than memorize, dread and fear mathematics.

Number Talks

One of the best methods for teaching number sense and math facts at the same time is a teaching strategy called ‘number talks’, developed by Ruth Parker and Kathy Richardson. This is an ideal short teaching activity that teachers can start lessons with or parents can do at home. It involves posing an abstract math problem such as 18 x 5 and asking students to solve the problem mentally. The teacher then collects the different methods and looks at why they work. For example a teacher may pose 18 x 5 and find that students solve the problem in these different ways:

20 x 5 = 100
2 x 5 = 10
100 - 10 = 90

10 x 5 = 50
8 x 5 = 40
50 + 40 = 90

18 x 5 = 9 x 10
9 x 10 = 90

18 x 2 = 36
2 x 36 = 72
18 + 72 = 90

9 x 5 = 45
45 x 2 = 90

Students love to give their different strategies and are usually completely engaged and fascinated by the different methods that emerge. Students learn mental math, they have opportunities to memorize math facts and they also develop conceptual understanding of numbers and of the arithmetic properties that are critical to success in algebra and beyond. Parents can use a similar strategy by asking for their children's methods and discussing the different methods that can be used. Two books, one by Cathy Humphreys and Ruth Parker (in press) and another by Sherry Parish (2014) illustrate many different number talks to work on with secondary and elementary students, respectively.

Research tells us that the best mathematics classrooms are those in which students learn number facts and number sense through engaging activities that focus on mathematical understanding rather than rote memorization.

The following five activities have been chosen to illustrate this principle; the appendix to this document provides a greater range of activities and links to other useful resources that will help students develop number sense.

Addition Fact Activities

Snap It: This is an activity that children can work on in groups. Each child makes a train of connecting cubes of a specified number. On the signal “Snap,” children break their trains into two parts and hold one hand behind their back. Children take turns going around the circle showing their remaining cubes. The other children work out the full number combination.
For example, if I have 8 cubes in my number train I could snap it and put 3 behind my back. I would show my group the remaining 5 cubes and they should be able to say that three are missing and that 5 and 3 make 8.

**How Many Are Hiding?** In this activity each child has the same number of cubes and a cup. They take turns hiding some of their cubes in the cup and showing the leftovers. Other children work out the answer to the question “How many are hiding,” and say the full number combination.

Example: I have 10 cubes and I decide to hide 4 in my cup. My group can see that I only have 6 cubes. Students should be able to say that I’m hiding 4 cubes and that 6 and 4 make 10.

**Multiplication Fact Activities**

**How Close to 100?** This game is played in partners. Two children share a blank 100 grid. The first partner rolls two number dice. The numbers that come up are the numbers the child uses to make an array on the 100 grid. They can put the array anywhere on the grid, but the goal is to fill up the grid to get it as full as possible. After the player draws the array on the grid, she writes in the number sentence that describes the grid. The game ends when both players have rolled the dice and cannot put any more arrays on the grid. How close to 100 can you get?

**Pepperoni Pizza:** In this game, children roll a dice twice. The first roll tells them how many pizzas to draw. The second roll tells them how many pepperonis to put on EACH pizza. Then they write the number sentence that will help them answer the question, “How many pepperonis in all?”

For example, I roll a dice and get 4 so I draw 4 big pizzas. I roll again and I get 3 so I put three pepperonis on each pizza. Then I write $4 \times 3 = 12$ and that tells me that there are 12 pepperonis in all.

**Math Cards**

Many parents use ‘flash cards’ as a way of encouraging the learning of math facts. These usually include 2 unhelpful practices – memorization without understanding and time pressure. In our Math Cards activity we have used the structure of cards, which children like, but we have moved the emphasis to number sense and the understanding of multiplication. The aim of the activity is to match cards with the same numerical answer, shown through different representations. Lay all the cards down on a table and ask children to take turns picking them; pick as many as they find with the same answer (shown through any representation). For example 9 and 4 can be shown with an area model, sets of objects such as dominoes, and the number sentence. When students match the cards they should explain how they know that the different cards are
This activity encourages an understanding of multiplication as well as rehearsal of math facts. A full set of cards is given in Appendix A.

![Multiplication Cards]

**Conclusion: Knowledge is Power**

The activities given above are illustrations of games and tasks in which students learn math facts at the same time as working on something they enjoy, rather than something they fear. The different activities also focus on the understanding of addition and multiplication, rather than blind memorization and this is critically important. Appendix A presents other suggested activities and references.

As educators we all share the goal of encouraging powerful mathematics learners who think carefully about mathematics as well as use numbers with fluency. But teachers and curriculum writers are often unable to access important research and this has meant that unproductive and counter-productive classroom practices continue. This short paper illustrates both the damage that is caused by the practices that often accompany the teaching of math facts – speed pressure, timed testing and blind memorization – as well as summarizes the research evidence of something very different – number sense. High achieving students use number sense and it is critical that lower achieving students, instead of working on drill and memorization, also learn to use numbers flexibly and conceptually. Memorization and timed testing stand in the way of number sense, giving students the impression that sense making is not important. We need to urgently reorient our teaching of early number and number sense in our mathematics teaching in the UK and the US. If we do not, then failure and drop out rates - already at record highs in both countries (National Numeracy, 2014; Silva & White, 2013) - will escalate. When we emphasize memorization and testing in the name of fluency we are harming children, we are risking the future of our ever-quantitative society and we are threatening the discipline of mathematics. We have the research knowledge we need to change this and to enable all children to be powerful mathematics learners. Now is the time to use it.
References


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## Appendix A: Further Activities and Resources

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How Close to 100?

You need
- two players
- two dice
- recording sheet (see next page)

This game is played in partners. Two children share a blank 100 grid. The first partner rolls two number dice. The numbers that come up are the numbers the child uses to make an array on the 100 grid. They can put the array anywhere on the grid, but the goal is to fill up the grid to get it as full as possible. After the player draws the array on the grid, she writes in the number sentence that describes the grid. The second player then rolls the dice, draws the number grid and records their number sentence. The game ends when both players have rolled the dice and cannot put any more arrays on the grid. How close to 100 can you get?

Variation
Each child can have their own number grid. Play moves forward to see who can get closest to 100.
How Close to 100?

1. ______ x ______ = ______
2. ______ x ______ = ______
3. ______ x ______ = ______
4. ______ x ______ = ______
5. ______ x ______ = ______
6. ______ x ______ = ______
7. ______ x ______ = ______
8. ______ x ______ = ______
9. ______ x ______ = ______
10. ______ x ______ = ______
Pepperoni Pizza

You will need
• one or more players
• 2 dice per player
• 10 or more snap cubes per player

In this game, children roll a dice twice. The first roll tells them how many pizzas to draw. The second roll tells them how many pepperonis to put on EACH pizza. Then they write the number sentence that will help them answer the question, “How many pepperonis in all?”

For example, I roll a dice and get 4 so I draw 4 big pizzas. I roll again and I get 3 so I put three pepperonis on each pizza. Then I write $4 \times 3 = 12$ and that tells me that there are 12 pepperonis in all.

Snap It

You will need
• one or more players
• 10 or more snap cubes per player

This is an activity that children can work on in groups. Each child makes a train of connecting cubes of a specified number. On the signal “Snap,” children break their trains into two parts and hold one hand behind their back. Children take turns going around the circle showing their remaining cubes. The other children work out the full number combination.
How Many Are Hiding

You will need
- one or more players
- 10 or more snap cubes /objects per player
- a cup for each player

In this activity each child has the same number of cubes and a cup. They take turns hiding some of their cubes in the cup and showing the leftovers. Other children work out the answer to the question “How many are hiding,” and say the full number combination.

Example: I have 10 cubes and I decide to hide 4 in my cup. My group can see that I only have 6 cubes. Students should be able to say that I’m hiding 4 cubes and that 6 and 4 make 10.

Shut the Box

You will need
- one or more players
- 2 dice
- paper and pencil

Write the numbers 1 through 9 in a horizontal row on the paper. Player 1 rolls the dice and calculates the sum of the two numbers. Player 1 then chooses to cross out numbers that have the same sum as what was calculated from the dice roll. If the numbers 7, 8 and 9 are all covered, player 1 may choose to roll one or two dice. If any of these numbers are still uncovered, the player must use both dice. Player 1 continues rolling dice, calculating the sum and crossing out numbers until they can no longer continue. If all numbers are crossed out the player say’s “shut the box”. If not all numbers are crossed out player 1 determines the sum of the numbers that are not crossed out and that is their score. If “shut the box” is achieved, player 1 records a score of “0”.

Player two writes the numbers 1 through 9 and follows the same rules as player 1. The player with the lowest score wins.

Variation
Player 1 and 2 can choose to play 5 rounds, totaling their score at the end of each round. The player with the lowest total score wins the game.
Math Cards

You will need

- one or more players
- 1 deck of cards (see next pages)

Many parents use ‘flash cards’ as a way of encouraging the learning of math facts. These usually include 2 unhelpful practices – memorization without understanding and time pressure. In our Math Cards activity we have used the structure of cards, which children like, but we have moved the emphasis to number sense and the understanding of multiplication. The aim of the activity is to match cards with the same numerical answer, shown through different representations. Lay all the cards down on a table and ask children to take turns picking them; pick as many as they find with the same answer (shown through any representation). For example 9 and 4 can be shown with an area model, sets of objects such as dominoes, and the number sentence. When students match the cards they should explain how they know that the different cards are equivalent. This activity encourages an understanding of multiplication as well as rehearsal of math facts.
$9 \times 4 = 36$

$6 \times 4 = 24$
4 × 6
6 × 4
$7 \times 6 = 42$
$9 \times 8 = 48$
Books:

By Jo Boaler


By Jo Boaler and Cathy Humphreys


Math Solutions - http://mathsolutions.com/

Math Solutions is a publishing company that has a range of excellent books to help parents and teachers with number sense

for example:


By Sherry Parrish


By Kathy Richardson


By Cathy Fosnot and Maarten Dolk

Fosnot, C., Dolk, M. (2001). Young Mathematicians at Work: Constructing Multiplication and Division:
Heinemann

By John Van De Walle and Lou Ann Lovin


By Heibert, Carpenter, Fennema, Fuson, Wearne and Murray


Additional Games:

Set http://www.setgame.com/set
Mancala

Games & Apps:

Mathbreakers https://www.mathbreakers.com
Motion Math http://motionmathgames.com/
Dragon Box http://www.dragonboxapp.com/
Refraction http://play.centerforgamescience.org/refraction/site/
Wuzzit Trouble http://innertubegames.net
Mancala http://www.coolmath-games.com/0-mancala/
Mary, a third grader, solves twelve minus five on her paper by crossing out the twelve and recording a zero above the ten and a twelve above the two. When asked to share why she solved the problem this way, Mary quickly replies, “Because you have to do it that way when the bottom number is bigger than the top number.”

We would like to believe that this is a unique situation; however, our classrooms are filled with students like Mary who view mathematics as a collection of rules and procedures to memorize instead of a system of relationships to investigate and understand (NRC 2001).
The math Process Standards highlighted in *Principles and Standards for School Mathematics* (NCTM 2000) and the National Research Council’s Strands of Mathematical Proficiency discussed in *Adding It Up* have encouraged mathematics instruction to move beyond rote procedural knowledge, but these instructional shifts have yet to be consistently embraced or reflected in student performance nationally or internationally (NRC 2001). The recently drafted Common Core State Standards (CCSS) continue to build on these processes and proficiencies with eight Mathematical Practices and calls for instruction grounded in conceptual understanding and mathematical reasoning (CCSSI 2010).

How can educators make shifts in their instructional practices that foster sense making in mathematics and move forward in developing mathematical dispositions as outlined in each of these documents? *Classroom number talks*, five- to fifteen-minute conversations around purposefully crafted computation problems, are a productive tool that can be incorporated into classroom instruction to combine the essential processes and habits of mind of doing math. During number talks, students are asked to communicate their thinking when presenting and justifying solutions to problems they solve mentally. These exchanges lead to the development of more accurate, efficient, and flexible strategies. What does it mean to compute accurately, efficiently, and flexibly? *Accuracy* denotes the ability to produce an accurate answer; *efficiency* denotes the ability to choose an appropriate, expedient strategy...
for a specific computation problem; and flexibility refers to the ability to use number relationships with ease in computation (Russell 2000). Take the problem $49 \times 5$ as an example. A student exhibits these characteristics if she can use the relationship between 49 and 50 to think about the problem as $50 \times 5$ and then subtract one group of 5 to arrive at the answer of 245. A more detailed example of how to develop these characteristics follows.

A classroom number talk
As I walk into Ms. Johnson’s fourth-grade classroom, the class is gathered on the floor discussing solutions for $16 \times 25$, a problem the students are solving mentally during their classroom number talk. Three student solutions are posted on the board for consideration: 230, 400, and 175. The soft hum of conversation can be heard as partners discuss their strategies and share their thinking. Johnson reminds the partners to discuss whether all answers are reasonable by quietly pointing to the class motto posted around the room: “Does it make sense?” She eavesdrops on their conversations until she has an understanding of their ideas and then brings students back for a whole-group number talk.

Please finish your last thought with your partner. Who would be willing to share their reasoning with the class?

[Madeline] Steven and I disagree with 230 and 175 because the answer has to be over 250. Why do you think the solution should be more than 250?

[Steven] Because $10 \times 25$ is 250, and you still have more to multiply. So we can rule out 230 and 175. [Most of the students reply.] Agree!

[Marquez] My partner and I can prove that 400 is the right answer.

How did you solve $25 \times 16$?
[Marquez] We knew that $4 \times 25$ was 100 and that there were four groups of $4 \times 25$ in $16 \times 25$. That would be the same as four 100s, which would equal 400.

Is this how you thought about the problem?
[Marquez] Yes. I knew I could break 16 into its factors of $4 \times 4$ and then I could multiply it in any order, but I used $4 \times 25$ first because that made a quick 100.

Why don’t you tell us how you thought about the problem.
[Anastasia] I got 230, but I don’t see why my answer isn’t right. I multiplied each part.

Why don’t you tell us how you thought about the problem.
[Anastasia] I multiplied $20 \times 10$ and that was 200, then I multiplied $5 \times 6$ and got 30. When I added those answers together, I got 230.

[Johnson scribes Anastasia’s strategy on the board for the students to consider.] [Jack] I don’t think you multiplied all the parts of the numbers; you missed some groups.

Let’s use an open array to help us think about Anastasia’s way and how she broke apart the numbers. [She draws an array representing $16 \times 25$ on the board.] “Anastasia, where did your $20 \times 10$ come from?”

[Anastasia] The 20 came from the 25, and the 10 came from the 16. I broke the 25 into a 20 and a 5 and the 16 into a 10 and a 6 and then multiplied $20 \times 10$ and $6 \times 5$.

[Johnson scribes Anastasia’s strategy on the open array, recording the multiplication expressions in the appropriate spaces in the array (see fig. 1).]

Nicole] I get it! She didn’t use all the groups! [Anastasia] Oh, I see! I left out the 10 groups of 5 and 6 groups of 20. If I add the 50 from $10 \times 5$ and the 120 from $6 \times 20$ plus my 230, I would get 400. [Jack] Amanda and I also got 400, and we multiplied every part, too. We did $6 \times 5$, $6 \times 20$, $10 \times 5$,
and $10 \times 20$; then we added all these answers to get 400.

[Paulette] Looking at the open array, I see why this way works. But it seems hard to keep track of all of the numbers unless you draw that.

[Louisa] Blakely and I also got 400, but we only broke apart the 16. We broke the 16 into 4, 2, and 10 and then multiplied each part by 25: $4 \times 25$ is 100, $2 \times 25$ is 50, and $10 \times 25$ is 250. All the parts added up to 400.

[Recording Louisa’s strategy] How do you know if you still have sixteen 25s?

[Louisa] Because $4 + 2 + 10$ is 16. We just broke 16 apart into easy numbers for us to multiply. We can use an open array to show why it works.

[As Louisa draws the corresponding array, Jack agrees that this is a more efficient way to multiply 16 × 25.]

[Stephanie] Michael and I doubled and halved. We doubled 25 to get 50, and then we halved 16 to get 8. All we had to do was multiply 8 times 50 to get 400. We know this works, because one side of the array doubles as the other side halves—the space inside the array stays the same.

[Several students exclaim at once] That’s really fast!

It looks like everyone is agreeing with 400, and we’re beginning to think about efficiency as well as accuracy [recording Stephanie’s strategy]. I’d like you to find a place to work with your partners to test each of these ideas [see fig. 2] to see if they would work for any multiplication problem.

The class tried several student strategies for $16 \times 25$ to see if they would work for any multiplication problem.

![FIGURE 2](https://www.nctm.org/teachingchildrenmathematics/issue/2011/2011-10-fig2.png)
A number talk’s key components
We can extract five essential components of a classroom number talk from Johnson’s classroom vignette: the classroom environment and community, classroom discussions, the teacher’s role, the role of mental math, and purposeful computation problems (Parrish 2010).

1. Classroom environment and community
Building a cohesive classroom community is essential for creating a safe, risk-free environment for effective number talks. Students should be comfortable in offering responses for discussion, questioning themselves and their peers, and investigating new strategies. The culture of the classroom should be one of acceptance based on a common quest for learning and understanding. It takes time to establish a community of learners built on mutual respect, but if you consistently set this expectation from the beginning, students will respond.

A first step toward establishing a respectful classroom learning community is acceptance of all ideas and answers—regardless of any obvious errors. Rich mathematical discussions cannot occur if this expectation is not in place. We must remember that wrong answers are often rooted in misconceptions, and unless these ideas are allowed to be brought to the forefront, we cannot help students confront their thinking. Students who are in a safe learning environment are willing to risk sharing an incorrect answer with their peers to grow mathematically.

Expecting acceptance of all ideas without evaluative comments is important. Educators can model this trait by recording all answers to be considered without giving any verbal or physical expressions that indicate agreement or disagreement with any answer. Teachers may need to practice having a “blank face.” Students look to teachers as the source of correct answers. Part of building a safe learning community is to shift this source of knowledge to the students by equipping them to defend the thinking behind their solutions.

2. Classroom discussions
A successful number talk is rooted in communication. During a number talk, the teacher writes a problem on the board and gives students time to mentally solve it. Students start with their fists held to their chests and indicate when they are ready with a solution by quietly raising a thumb. Once students have found an answer, they are encouraged to continue finding efficient strategies while others are thinking. They
indicate that they have found other approaches by raising another finger for each solution. This quiet form of acknowledgment allows time for students to think, while the process continues to challenge those who already have an answer. When most of the students have indicated they have a solution and a strategy, the teacher calls for answers. All answers—correct and incorrect—are recorded on the board for students to consider.

The benefits of sharing and discussing computation strategies are highlighted below. Students have the opportunity to do the following:

- Clarify thinking.
- Investigate and apply mathematical relationships.
- Build a repertoire of efficient strategies.
- Make decisions about choosing efficient strategies for specific problems.
- Consider and test other strategies to see if they are mathematically logical.

Number talks may naturally lead to student investigations of strategies, as we saw at the conclusion of Johnson's number talk. Students can test numerous problems using strategies that surface during the number talk. By keeping a record of which strategies work and under what parameters, they can share their findings with the class and make a group decision on whether the strategy is mathematically logical, able to be generalized, and can be applied in all situations. This is another way to transfer the ownership of learning to students. We can see how this might unfold in the classroom by continuing to follow the conversations with Johnson's students after they have investigated strategies exhibited during their class number talk.

I noticed many of you chose to investigate the doubling-and-halving strategy that Stephanie and Michael used in our number talk. Would someone share what you discovered?

[Jake] Marquez and I first tried doubling and halving by testing smaller numbers like $6 \times 9$, $8 \times 14$, and $5 \times 15$; it worked with the first two problems but not with $5 \times 15$.

Why do you think it didn't work with $5 \times 15$?

[Marquez] We think it's because 5 and 15 are odd numbers, and you can't halve an odd number.

[Blakely] Yes, you can. If you halve 5, you get 2 and a half, and half of 15 is 7 and a half.

[Anastasia] I agree with Blakely; you could do it, but it wouldn't necessarily make it an easier way to solve the problem.

[Desmond] Nicole and I also tried doubling and halving with two odd numbers, $9 \times 13$, but we ended up with fractions, and that made the problem harder to solve.

If doubling and halving isn't necessarily an efficient strategy when multiplying two odd numbers, when is it an efficient method?

[Marilyn] Josh and I tested problems when both numbers were even and also when one number was even and one was odd, but it seemed to make the problems easier to solve when one factor was even and the other was odd.

Can you give us an example?

[Marilyn] First we tried just even numbers with $12 \times 16$, and then doubled and halved to change the problem to $6 \times 32$, then $3 \times 64$. This made it a one-digit times a two-digit number, but it still wasn't a simple problem. We even tried doubling the 12 factor and halving the 16, but we ended up with $96 \times 2$. Next, we tried an odd number times an even number, and it seemed easier. We did $12 \times 15$ and doubled and halved to get $6 \times 30$, then $3 \times 60$; it was fast and efficient.

[Roberto] Juan and I found the same thing—it seems like a more efficient way when you have an odd number multiplied with an even number.

I'm seeing lots of nods of agreement with this statement. I'd like us to keep investigating to see if our generalization holds true and if there are any exceptions to our rule.

3. The teacher's role

As educators, we are accustomed to assuming the roles of telling and explaining. Teaching by telling is the method many of us experienced...
as students, and we may have a tendency to emulate this model in our own practice. Because a primary goal of number talks is to help students make sense of mathematics by building on mathematical relationships, our role must shift from being the sole authority in imparting information and confirming correct answers to assuming the interrelated roles of facilitator, questioner, listener, and learner.

Since the heart of number talks is classroom conversations focused on making sense of mathematics, it is appropriate for the teacher to move into the role of facilitator. Keeping the discussion focused on the important mathematics and helping students learn to structure their comments and wonderings during a number talk is essential to ensuring that the conversation flows in a natural, meaningful manner. As a facilitator, you must guide students to ponder and discuss examples that build on your purposes. By posing such questions as, How does Joey’s strategy connect to the ideas in Renee’s strategy? you lead conversations to build on meaningful mathematics.

As we move toward listening to our students’ thinking instead of concentrating on only a final, correct answer and one procedure, we will begin to ask open-ended questions. By changing our question from, What answer did you get? to How did you solve this problem? we will be able to understand how students are making sense of the mathematics.

4. Role of mental math
If students’ math experiences have primarily focused on learning and practicing the standard U.S. algorithms for each operation, they may be resistant to looking at problems from other perspectives. Some students may try to visualize the problems vertically and will even write the problem with their fingers on the floor or in the air to remain consistent with the paper-and-pencil algorithms they may have learned. Mental computation is a key component of number talks because it encourages students to build on number relationships to solve problems instead of relying on memorized procedures. One purpose of a number talk is for students to focus on number relationships and use these relationships to develop efficient, flexible strategies with accuracy. When students approach problems without paper and pencil, they are encouraged to rely on what they know and understand about the numbers and how they are interrelated. Mental computation encourages them to be efficient with the numbers to avoid holding numerous quantities in their heads.

Mental computation also helps strengthen students’ understanding of place value. By looking at numbers as whole quantities instead of discrete columns of digits, students must use their knowledge and understanding of place value. During initial number talks, problems are often written in horizontal format to encourage student's thinking in this realm. A problem such as 199 + 199 helps illustrate this reasoning. By writing this problem horizontally, you encourage a student to think about and use the value of the entire number. A student with a strong sense of number and place value should be able to consider that 199 is close to 200; therefore, 200 + 200 is 400 minus the two extra units for a final answer of 398.

Recording this same problem in a vertical format can encourage students to ignore the magnitude of each digit and its place value. A student who sees each column as a column of units would not be using real place values in the numbers if they are thinking about 9 + 9, 9 + 9, and 1 + 1.

5. Purposeful computation problems
Crafting problems that guide students to focus on mathematical relationships is an essential part of number talks that is used to build mathematical understanding and knowledge. The teacher’s goals and purposes for the number talk should determine the numbers and operations that are chosen. Carefully planning before the number talk is necessary to design “just right” problems for students.
For example, if the goal is to help students develop multiplication strategies that build on using tens, starting with numbers multiplied by 10 followed by problems with 9 in the units column creates a situation where this type of strategy is important. Such problems as $20 \times 4$, $19 \times 4$, $30 \times 3$, $29 \times 3$, $40 \times 6$, and $39 \times 6$ lend themselves to strategies where students use tens as friendly, or landmark, numbers. In the problem $19 \times 4$, the goal would be for students to think about $20 \times 4$ and subtract 4 from the product of 80, because they added on one extra group of 4. Other problem sets can then be designed to elicit a similar approach. In later number talks, the teacher would begin with a number with 9 in the units column without starting with the multiple of 10 (see fig. 3).

Does this mean that given a well-crafted series of problems, students will always develop strategies that align with the teacher’s purposes? No. Numerous strategies exist for any given problem; however, specific types of problems typically elicit certain strategies. Take, for instance, the same $19 \times 4$ problem that was crafted to target students’ thinking using tens. Students could approach this problem in a variety of ways, including but not limited to the following:

<table>
<thead>
<tr>
<th>Focus on using doubling and halving in multiplication</th>
<th>Focus on using landmark numbers in multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \times 48$</td>
<td>$3 \times 50$</td>
</tr>
<tr>
<td>$2 \times 24$</td>
<td>$3 \times 49$</td>
</tr>
<tr>
<td>$4 \times 12$</td>
<td>$5 \times 200$</td>
</tr>
<tr>
<td>$8 \times 6$</td>
<td>$5 \times 199$</td>
</tr>
<tr>
<td>$16 \times 3$</td>
<td></td>
</tr>
</tbody>
</table>

FIGURE 3

Shown are examples of purposefully designed computation problems.
Break the 19 into 10 + 9; then multiply 10 × 4 and 9 × 4 and combine these products.

Break the 4 into 2 + 2; then multiply 2 × 19 and 2 × 19 and combine these products.

Add 19 + 19 + 19 + 19, or add 4 nineteen times.

However, a mixture of random problems, such as 39 × 5, 65 – 18, and 148 + 324, do not lend themselves to a common strategy. This series of problems may be used as practice for mental computation, but it does not initiate a common focus for a number talk discussion.

Taking the first steps

Making the shift toward teaching for understanding by encouraging students to develop mental math strategies can often be overwhelming. Many of us are comfortable with telling students how to solve a problem but avoid focusing on student-invented strategies because doing so may feel foreign and intimidating. As you begin implementing number talks, consider starting first with smaller numbers, such as basic facts. Using basic facts as a starting place is an excellent way to establish that many ways exist to view and approach a problem. With the fact 6 + 7, we see multiple ways for students to think about this problem:

- **Use** doubles (6 + 6 = 12 plus one more; 7 + 7 = 14 minus one).
- **Make** a quick ten (6 can be split into 3 + 3 and 3 + 7 = 10 plus three more).
- **Count on** or count all (7, 8, 9, 10, 11, 12, 13).

Give yourself the license to be a learner along with your students and to question, “Does it make sense?” When students become accurate, efficient, and flexible with their mental math strategies, you can then transition to their regular paper-and-pencil computation practice. Encourage them to incorporate their mental math strategies from number talks with other algorithms by solving each problem in two different ways. This will not only help serve as a system of checks and balances for accuracy but also help students develop and maintain flexibility in thinking about numbers.

REFERENCES


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Number Talks

Think!
- No Voices
- Persevere... try your best
- Mental Strategies

Thumbs Up
- I have a total
- I have a strategy

Share Totals
- Same total
- Same strategy

Construct Arguments
- I noticed...
- I was wondering...
- My strategy was...
- How did you solve?
- My thinking...
- My strategy was similar...

Critique Reasoning
- My strategy was different...